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ON THE MEASUREMENT OF THE EXPANSION OF METALS BY THE INTERFERENTIAL METHOD.

By Edward W. Morley and William A. Rogers.

Note by William A. Rogers.

During the summer of the year 1890 Professor Morley suggested to the writer the method, described in the paper herewith presented, for determining the coefficient of expansion of a bar of metal between the freezing and the boiling points of water in terms of a wave-length of light of known refrangibility.

After a full discussion of the form of apparatus best adapted for this purpose, it was arranged that I should construct a simplified form which should suffice for some preliminary experiments; that Professor Morley should come to Waterville during his summer vacation, and that we should jointly undertake the experiment.

The first series of observations was made in July, 1891. Both of the bars selected, one of steel and the other of bronze, were observed continuously for a period of twelve hours in melting ice. During this time the relative lengths of the two bars suffered no perceptible change. The ice which surrounded the steel bar was then removed, and the bar was kept in steam for several hours, the bronze bar remaining in melting ice. Under this condition the difference in the lengths of the two bars was also measured.

In this experiment, however, as was foreseen, it was found to be impossible to count the number of fringes of the sodium light em-

ployed, on account of the difficulty of obtaining sufficient freedom from the condensation of moisture upon the mirrors during the passage from low to high temperatures. The only thing which could be done under these circumstances was to measure the difference in length by means of a microscope attached to a plate upon which the steel bar was mounted, the graduated scale employed being disconnected from this plate.

This experiment confirmed our previous impression that all observations of this character must be made *in vacuo*.

It was therefore found necessary to reconstruct the refractometer. Profiting by the experience already gained, several modifications were made in the construction of the new apparatus. The bars compared were mounted in boxes of rolled brass. It was supposed that the air would be unable to enter the boxes through the pores of this metal. It was found, however, impossible to maintain a vacuum during the time required for a complete observation. After a fruitless trial of fourteen months it was decided to commence *de novo*.

The two brass boxes and the box for receiving the diagonal mirrors were cast in one piece, which weighed about 500 lbs. Every part of this casting was then covered on the outside with solder to the depth of one-eighth of an inch. The work was done with the ordinary soldering iron. The magnitude of this undertaking may be inferred when it is stated that the boxes with their connections are nearly 60 inches in length and 6×6 inches in cross-section. One hundred and fifty pounds of solder were required for this purpose. The construction of this refractometer was completed in 1894. Several months were occupied in testing the apparatus in every possible way. Early in June, 1895, I wrote to Professor Morley that, as far as I could see, everything was ready for the final trial. He therefore came to Waterville early in July for this purpose.

While the two experiments recorded in this paper are not in every respect satisfactory, they demonstrate with entire certainty the practical feasibility of the new method. In the first series of observations, a little oil was forced through the packing and dropped upon one of the mirrors when the temperature of 65° C.

was reached, thus compelling the abandonment of the experiment at this point.

In the second series, after the measures had been made at 65°, the vapor of sulphur extracted from the rubber packing tarnished the mirrors to such an extent that while the fringes could very easily be counted near the points of maximum visibility, it was found impossible to make the count at the minimum point beyond the second order. It was apparent, however, that there would be no difficulty in making the count at 100°, after freedom from the sulphur vapor should be secured.

In another respect the result of this experiment was a little disappointing. Throughout the observations the readings for temperature were made with the air thermometer; but at the very last moment it was found that, owing to a mechanical defect, which could not have been known until after the reduction of the observations, the readings taken could not be depended upon. It therefore became necessary to make a careful investigation of all the thermometers employed in the experiment. This discussion is given in the second part of this paper. The general features of the different forms of the apparatus employed in this investigation will be seen from the diagram and photographic views shown in the plate accompanying Part II.

SHANNON PHYSICAL LABORATORY, COLBY UNIVERSITY,
March 1, 1896.

PART I.

OPTICAL PRINCIPLES ON WHICH THE METHOD DEPENDS.

BY EDWARD W. MORLEY.

Two methods for determining the expansion of metallic bars are in common use. In the first method, micrometers are employed to measure, at different temperatures, the distance between two lines cut near the ends of the bars in question. If the bars compared are both exposed to the same temperatures, the meas-

urements determine the difference between their expansions; but if one bar is kept at a constant temperature, while that of the other is varied, the measurements determine the absolute expansion of the latter.

In the second method, the changes in the length of a bar exposed successively to different temperatures are determined by interferential observations. This method gives, not absolute expansions, but the difference between the expansion of the bar and that of the support of the interferential apparatus, which can be previously determined by interferential observations of great accuracy. The method has been used only for bars of but a few millimeters in length.

The accuracy of which these methods are capable depends on the precision of measurements of length and of temperature. The International Bureau of Weights and Measures estimates that coefficients of expansion may be determined within one part in twelve hundred.

A method which was first suggested some eight years ago, and which seems likely to attain a somewhat greater accuracy, has now been put in practice, although with some of the imperfections to be expected in a first attempt. The results obtained are of such a character that it is now proper to describe the method more fully than has been done heretofore.

The new method resembles, in certain respects, each of the former methods. It resembles the micrometric method, for bars of any length may be used, and their lengths are compared while the temperature of one is varied, and that of the other is either varied or kept constant. It resembles Fizeau's method, for interferential phenomena are utilized. But the new process differs so materially from both of the older processes, that it cannot fairly be called a modification of either. Interferential observations replace the micrometric observations of the first process; but they differ both in method and in principle from those of Fizeau, for they are used, just like micrometric observations, to measure the difference of the length of two bars.

We have, in this interferential method of comparison, much greater accuracy in measuring small differences of length. We

are also able, incidentally, to place the bars, during the comparison, in conditions especially favorable for accurate determination of their temperature. It is therefore fair to expect ultimately a decided gain in accuracy of measurements of expansion.

To explain the principle of this method, let *ab*, Fig. 1, represent a plane mirror, whose thickness we may for a moment imagine to be negligible. Suppose this mirror to be a film of silver of such thickness that thirty or forty per centum of the light falling on it is transmitted, and the remainder reflected. Let *c* and *d* be the axes of two metal bars ending in points; let the bars and the mirror be so adjusted that the image of *c*, seen by the eye placed at *k*, is parallel with *d*, and near it. Let *e*, *f*, *g*, *h*, be four plane mirrors, each held by a spring against one of the points which terminate the bars, and also against the points of two adjusting screws, by means of which we make the four mirrors perpendicular to the axes of the bars. Let means be provided for moving the bar *d*, with its mirrors, in the direction of its length. For the present, imagine the thickness of the mirrors *e* and *g* to be negligible.

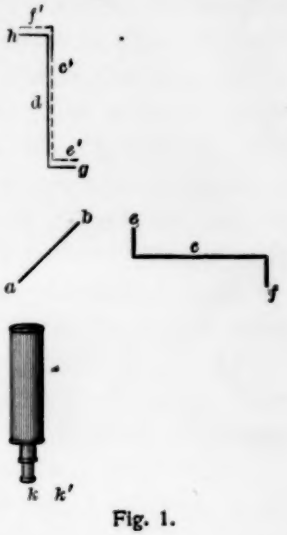


Fig. 1.

If now a source of light is placed to the left of *ab*, part of a beam falling on *ab* will be transmitted to *e* and *f*, most of this fraction will be there reflected back to *ab*, and in part to the eye at *k*. If we suppose, for example, that *ab* reflects seventy per centum of the light falling on it, and suppose no loss to occur in the reflections at *e*, *f*, *g*, *h*, and none by absorption in *ab*, twenty-one per centum of the light falling on *ab* will reach the eye at *k* by the route just mentioned. On the other hand, seventy per centum of the light incident on *ab* will be reflected to the mirrors *g* and *h*, and twenty-one per centum will be transmitted through *ab* to *k*. The transmitted and reflected rays will therefore be of equal intensity when they reach the eye, and will be in condition to produce interference phenomena of maximum distinctness.

It is obvious that the effect of the thinly silvered mirror *ab* is to superpose, optically, the system *ecf* upon the system *gdh*. To save circumlocution, let us represent the *apparent* position of *ecf* by the accented letters $e'c'f'$.

If the source of light is monochromatic, and if the distances of g and h from the eye at k do not differ too much from those of e' and f', interference fringes will be seen by the eye at k. When f' and h are not equidistant from k, these fringes will have their maximum distinctness if f' and h are very nearly parallel; the interference fringes will then be seen as circles when the observing telescope is adjusted for parallel rays. The same is true for e' and g. Suppose that such parallelism of f' and h, as well as of e' and g, is obtained and is preserved while we slowly move the system *gdh* so as to bring h nearer to f'. Each of the circular fringes seen at k will enlarge and disappear, while new fringes will appear at the center of the system of circular fringes. As f' and h approach, the number of circles which can be seen at once diminishes; at last, the field of view contains but a fraction of the diameter of the two inner fringes, and the continued approach of f' and h produces simple alternations of light and darkness. If, at the right time, white light is substituted for monochromatic, alternations of color will begin to be seen, till f' and h coincide, when we shall perceive the darkness which is produced by two equal beams of white light whose paths have been optically equivalent while one of them has suffered the change of phase which is due to an internal reflection.

Now, when the planes f' and h coincide, it is obvious that the ends of the axes of c' and d are in the same plane perpendicular to these axes. At this moment let us restore the monochromatic light, move the telescope to the position k', and observe the interference phenomena at the surfaces e' and g. It is obvious that, if e' and g are very nearly parallel, their distance is equal to the difference in the length of the two bars. We can move d in the direction of its length till e' and g coincide, and we can determine when they coincide by the appearance of the central black fringe in white light. Further, if we can determine how far d has to be moved to produce alternate coincidence of f' with h, and of e'

with g, we shall have a measure of the difference of length of c' and d.

If this difference is not more than one or two tenths of a millimeter, a convenient method is to count the number of half-wave-lengths contained in it, and to measure the residual fraction. For such small differences, the new method depends on such direct counting, but for greater differences, another principle is to be used.

The mirrors e' and g, f' and h, have been supposed to be rigorously parallel, because, in this case, the interference phenomena have the maximum of distinctness, and because the geometrical relations are simple. But another adjustment of the mirrors is commonly more convenient in practice. For this second adjustment of the mirrors, it is necessary that the axes c' and d should very nearly coincide. If such coincidence is secured, and if we are looking at $e'c'f'$ and gdh from one side, we should see them as in Fig. 2, supposing that $e'c'f'$ were a real object and not an image. Now let e' and f', still touching the points of c', be rotated a minute of arc, or less, about an axis in the plane of Fig. 1, and perpendicular to the plane of Fig. 2, so as to take the position shown in Fig. 3. The interference fringes will now appear as lines; these are straight, and of maximum distinctness when e' and f' are equidistant from the eye; they are also sufficiently distinct with the greatest difference of distance which is required in measuring the expansion of bars not more than one or two meters long.

Figs. 2 and 3.

With this adjustment, and with monochromatic light, we see the interference fringes when the telescope is adjusted for distinct vision of the surfaces of the mirrors e', g, or f', h, respectively. Suppose we are observing f', h, Fig. 1; suppose also that the upper half of f' was tilted backwards; if we slowly bring h nearer to f', the interference fringes, which appear as horizontal lines, will actually move upwards, and will seem to move downwards as seen in an inverting telescope. When the lower edges of f' and h are within about ten half-wave-lengths, colored fringes will

begin to be seen in white light; when these lower edges coincide, the central black fringe will be there seen in white light. As the motion of *d* continues, this central fringe will move upwards (apparently downwards); when it passes through that point of the mirror *f* which rests on the point of the bar *c'*, the end of *c'* coincides with the plane *h*, and with the end of the bar *d*. Let us now move the telescope to the position *k'*, and obtain distinct vision of the surfaces *e'* and *g*; then, if the distance by which *d* must be moved to make the remaining ends of *c'* and *d* coincide is a whole number of half-wave-lengths of the monochromatic light used, a black fringe will pass through that point of the mirror *e'* which rests upon the point of the bar *c'*. But if this distance is a whole number, together with some fraction, the magnitude of this fraction is determined by micrometric or other measurement; after which, the whole number is observed by slowly moving *d*, and counting the number of fringes which pass a mark ruled on the surface of the mirror. We shall have thus determined the difference of length of *c* and *d*, as expressed in half-wave-lengths of the monochromatic light used.

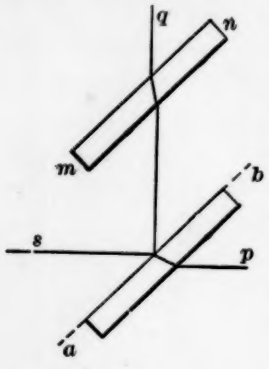


Fig. 4.

The thickness of the mirrors has been neglected in this explanation, and must now be considered. The mirror *ab*, Fig. 1, consists of a thin layer of silver deposited on the surface of a plane parallel plate of optical glass, *ab*, Fig. 4. A second plate, *mn*, of equal thickness, cut from the same disk, is set parallel to *ab*. The rays *sp* and *sq* pass through the same thickness of glass; if they also pass through the same thickness of air, at the same temperature, they will have equivalent optical paths.

The mirrors *f* and *h*, Fig. 1, have their reflecting surfaces supported on the points which determine the effective lengths of the bars. Their thickness, therefore, has no effect. But, with the constructions so far used in actual experiment, the mirrors *e* and *g* are supported on one surface while they reflect light from the

other. Now these mirrors are plane-parallel plates of glass; if their temperatures are the same, and their thickness is the same, their effects will cancel each other. If their temperatures are the same, but their thicknesses different, their effects may be made to cancel by reversal. But if their temperatures are different, the coefficient of expansion of one of them must be known.

Two constructions are possible in which the expansion of the glass of the mirror need not be known. In one of these, both mirrors on the bar whose temperature is varied are supported by the reflecting surface. But this involves the cutting of a notch in the edge of the bar, and so is limited in application.

We have tried various methods for supporting the mirrors perpendicular to the axes of the bars, and have others in mind.

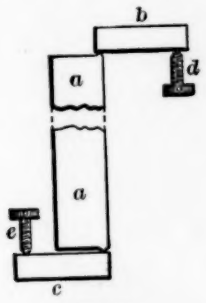


Fig. 5.

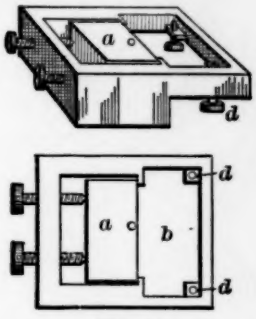
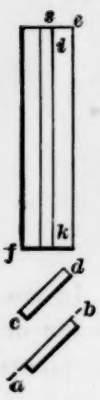


Fig. 6.

These vary according to the purpose of the experiment. In one form, convenient when many measurements have to be made on the same pair of bars, the mirrors *b* and *c*, Fig. 5, rest on points on the bar *aa*, and also upon screws *de*, which are carried on pieces clamped to the bar. One such piece is shown in Fig. 6. Springs which press the mirror against the supporting points are also attached to the same piece; so that the mirrors, the adjusting screws, and the springs are somewhat permanently associated, and may be handled or adjusted as a whole. It is then convenient to support the bar by means of a hole, a groove, and a plane, resting on three points; in which case, the bar and its mirrors may be removed from the interferential comparer and again replaced without needing readjustment.

When the mirrors are supported in this way, they are adjusted so as to be perpendicular to the axis of the bar by the use of a collimating telescope, in a way which is perhaps too simple to need description. The two bars and their mirrors are then placed in the interferential comparer, the mirrors of the bar which is to be kept at constant temperature are left undisturbed throughout the experiment, while the mirrors on the other bar are given the slight inclination needed to make the direction and width of the interference fringes convenient for observation.

When the mirrors are supported and adjusted as now described, the subsequent adjustment of the bars for coincidence of the axes c' and d, Fig. 1, is somewhat troublesome. If experiments are to be made on a number of different bars in succession, it may be well to facilitate this adjustment of the axes in coincidence. In the method used for this purpose, the bars are first put in position in the interferential comparer, so that the lines joining the points on which the mirrors are to be supported, are made to coincide; after which the mirrors are brought up to the ends of the bars, and made perpendicular to them.



For this purpose, the metal base of the apparatus has two grooves, *ik*, *em*, Fig. 7. In these grooves are fitted four pieces *oo*, Fig. 8, each having a round aperture with cross-lines *p*. These pieces and the apparatus are adjusted so that, if two pieces are placed at *i* and *e*, Fig. 7, with lights at *r* and *s*, the two sets of cross-wires coincide, one seen direct, and one reflected at the surface *ab*. Two other pieces set at *k* and *m* are, in like manner, made to coincide. These four pieces are marked so as always to be used in the same positions.

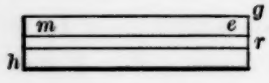


Fig. 7.

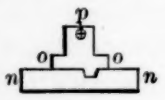
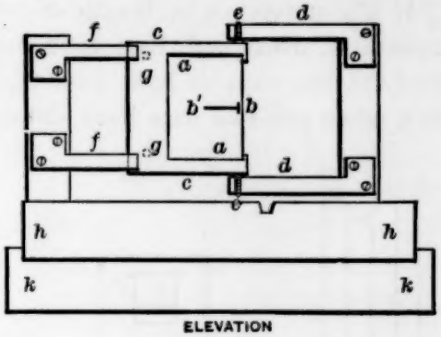


Fig. 8.

With this preparation, we can readily put two bars into the required position. We replace the pieces at *i* and *k*, and then adjust the bar with the points at its ends in the position shown by the cross-wires at *i* and *k*; we do the same with the other bar and the cross-wires at *m* and *e*, when the axes c' and *d*, Fig. 1, will coincide.

The mirrors may now be applied to the ends of the bars. The mirrors are held as shown in elevation by Fig. 9, and in plan by Fig. 10. The mirror *a* is contained in a brass frame *cc*. On this frame are two pivots, *ee*, which turn in holes made in the ends of the springs *dd*, which press the mirror against *b*, the point of the bar, whenever the mirror and its supports are brought sufficiently near the bar. Other springs, *ff*, press the frame against adjusting screws *gg*.



ELEVATION

Fig. 9.

During the adjustment of the bars to the proper position with respect to each other, the mirrors are slipped back twenty or thirty millimeters from the ends of the bar, so that the pieces *o*, Fig. 8, can be put at *i*, *k*, *e*, *m*, Fig. 7, at the ends of the bar. When the bar has been adjusted to the position shown by the cross-wires, and secured there, these pieces are removed, and the mirrors and their supports are moved up to the ends of the bar by sliding the base *h*, Figs. 9 and 10, in the ways which carry it. When they have been clamped, the screws *g* are used to make the mirrors very nearly perpendicular to the axes of the bars by means of a collimating telescope which has been previously adjusted to the required position.

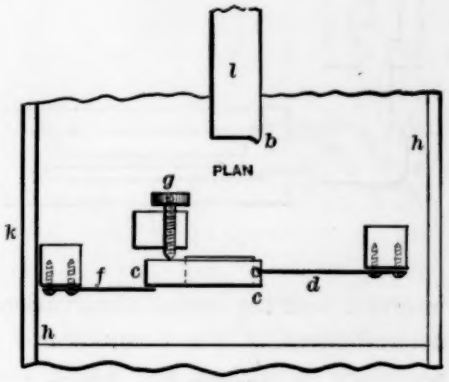


Fig. 10.

On the mirrors, lines *bb'*, Fig. 9, may be ruled, so as to mark the points of contact of the mirrors and the points of the bars. When the central black interference band in white light is brought to coincide with *bb'*, the end of one bar coincides with the image

of the other bar within limits which it is easy to make less than the thirty-thousandth of a millimeter.

Neither of these methods of supporting the mirrors is the best; but it will be best to explain some other points before describing the method which will probably be found most accurate as well as convenient when we shall have had time to prepare suitable apparatus.

If the difference of length of two bars is determined in this apparatus, when both bars have the same temperature, the fact that the two rays of light pass through air introduces no error. But when the two bars have different temperatures, the optical path of the ray near the cold bar is made too great by the greater refractive index of the colder air. For this error, a suitable numerical correction could be applied. But, with any convenient arrangement of the apparatus, the presence of air produces another difficulty, which is more serious. Irregular currents are set up by the difference of density of the hot and the cold air, which interfere with the optical observations as well as with the accurate determination of the temperatures of the bars. It is therefore necessary to inclose the whole optical apparatus in a suitable chamber from which the air can be removed.

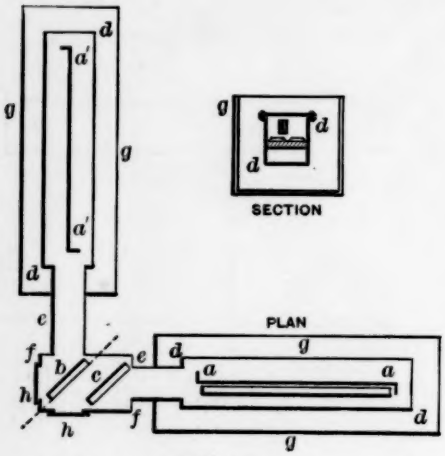


Fig. 11.

The arrangement used in the experiments which we have made is shown in Fig. 11. *aa*, *a'a'*, are the bars and their mirrors; *b*, *c*, are the diagonal mirror and its compensation. The bars are contained in two brass boxes *dd*, *dd*, one of which is also shown in section. These are placed in larger iron boxes *gg*, designed to hold water, ice, or steam, for maintaining the desired temperature. Tubes *ee* connect *dd*, *dd*, to the box *ff*, which contains the

diagonal mirror and its compensation. *hh* are two openings covered with glass; through one light is admitted to the apparatus, and the observing telescope is placed before the other.

A screw, not shown in the drawings, provides the means of moving the bar $a'a'$ in the direction of its length. The screws for adjusting the mirrors perpendicular to the axes of the corresponding bars are connected to handles which pass through stuffing boxes in the air-tight case.

It is obvious that, if we desire to measure expansions more accurately than to one part in one thousand, we must be prepared to measure differences of temperature with corresponding accuracy. Now, in our present apparatus, the bar $a'a'$ is to be kept at a constant temperature by means of ice. We have accordingly thought it sufficient, at least in preliminary experiments, simply to verify the constancy of its temperature by means of delicate mercurial thermometers. These thermometers, each graduated in fiftieths of a degree, are introduced through the cover of the box; their bulbs rest on the bar $a'a'$, while their scales are in position for convenient observation.

The bar *aa* is that whose expansion is to be measured; its temperature must therefore be determined with accuracy. In our present apparatus, a square brass tube, about a meter long and twenty-five millimeters in diameter, is used as the bulb of an air-thermometer, and is placed in contact with the bar *aa*, to which it may well be attached by tying with wire. A capillary brass tube connects this thermometric reservoir with a suitable apparatus for measuring the pressure of the inclosed hydrogen.

When we desire to measure the difference of length of the two bars at some temperature which can be maintained constant by some physical process, like that of melting ice, we can make sure that the bars have actually reached a constant temperature, by observing whether the interference phenomena have become stationary. When the thermometers have also become stationary in their indications, we know that the thermometers indicate the temperature of the bars, for the bars, the thermometers, and their envelope are all at the same temperature, and no air is present. Interferential observations of the difference of length of the bars

can then be begun, carried on, or repeated, with a degree of convenience amounting almost to luxury. The case is nearly the same if the temperature of boiling water is maintained by the use of steam; variations due to change of barometric pressure produce no inconvenience. But when some intermediate temperature is to be maintained by the use of water heated by leading steam into it, we have constantly to observe the temperature of the water by means of a delicate thermometer immersed in it. This involves vigorous and constant agitation of the water, during which observation of interferential phenomena is impossible on account of vibration. If, therefore, for the determination of the difference of length of two bars at a given temperature, this temperature had to be maintained during the whole operation, the new method could be applied only at temperatures which can be procured by utilizing melting or boiling points.

But, fortunately, the known and measured temperature is required for only a few seconds during the measurement. To illustrate from a simple operation, suppose we desire to measure the length of a steel tape. We have to uncoil it, and lay it on the scale *ab*, Fig. 12, and to keep it stretched with a known tension while we make the end *c* coincide with *a*, and while we then transfer our attention to the point *d* and press the other end of the tape against the scale. During the moment of this transfer of attention, we need to know the tension of the tape; but then, provided we hold *d* in its position, we may, if we please, let the tape be coiled again; we have nothing more to do with the end *c*, nor with the tension of the whole.

So with the interferential measurement. We interrupt the stirring of the water-bath surrounding the bar long enough to verify that *a* and *c*, Fig. 12, coincide; a matter of ten seconds or less. Repeating this verification at intervals, we may still maintain the temperature of the water-bath unchanged; during this time, the air thermometer is read. Now the observer at the telescope transfers his attention to *d*, and the other observer makes his final reading of the air thermometer. After this moment, nothing can interfere with the

Fig. 12.

accuracy of the interferential measurement of the distance *bd*, except some accident which interrupts the observations.

It may be said that, in our apparatus, the temperature of the bar *aa*, Fig. 11, can be observed by means of three thermometers, graduated in fiftieths of a degree, whose bulbs are in contact with the bar. These are useful for procuring and maintaining the desired temperature, and for verifying its constancy; but, in accurate observations, the temperature of the bar must be learned from the air thermometer whose reservoir is in contact with the bar.

In our apparatus, the motion of the bar $a'a'$, Fig. 11, is produced by a screw. This screw is turned by a worm-gear and worm, so that a motion of a small fraction of a wave-length can easily be procured. The experiments have been much delayed by the difficulty we found in getting brass boxes *dd*, *ff*, Fig. 11, which should not leak. One set had to be rejected after the apparatus was completed. A second set has shown leakage only in the stuffing boxes which surround the thermometers and the rods by which the adjustment of the mirrors is accomplished, and this leakage has been so far controlled that experiments have been successfully made with the apparatus.

In the experiments which have been made so far, the expansion of the bar has been determined by counting the whole number of wave-lengths which measures this expansion; the number was nearly 3500. To count 500 wave-lengths is easy, and takes less than ten minutes. But to count 3500 cannot be pronounced easy. We have commonly used the monochromatic light produced by putting a sodium compound in a colorless gas flame. We have sometimes used the green light produced by mercurial vapor in a Plücker's tube, and had no difficulty in making this so bright that we could count wave-lengths in a room where direct sunlight fell near the observer. But, with the means at our disposal, it was not easy to keep the induction coil working well for the period required, so that we returned to the use of the sodium light. Now, owing to the fact that this light is compounded of light of two wave-lengths which differ by about one thousandth part, the distinctness of the interference bands produced by it

suffers periodic variations. If we begin to count when the two interfering rays have traversed equivalent paths, then the bands numbered from 450 to 550 are obscure, those from 1450 to 1550 and those from 2450 to 2550 are more obscure, and those from 3450 to 3550 are so obscure as to be very difficult to count. *This* difficulty in counting can readily be overcome by using the green light of incandescent mercury vapor, in which case no variations of distinctness in the interference bands occur within the limits utilized in observations on expansion. Suitable methods of breaking the circuit of the induction coil will make its action sufficiently uniform for many hours;[1] and Plücker tubes containing mercury can be so constructed that they will not fail by leakage due to heating the electrodes.

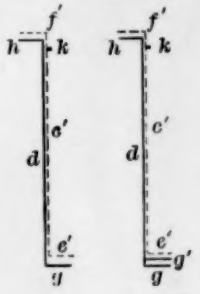
But even with every convenience for counting 4000 wave-lengths, the operation can hardly occupy less than an hour; so that observations at the temperatures which require the counting of such large numbers could not be readily multiplied. But, *during the actual measurement of expansions*, no count longer than 400 is necessary; in fact, it is possible to reduce the counting required, *during the actual comparison of the length* of two bars, to a hundred, to fifty, or even to twenty wave-lengths.

The methods devised with this end in view form a part of the original plan, which has not yet been carried out. But all the optical, and most of the mechanical, details of the method rest upon experiment, so that no uncertainty is felt as to the result. In order to explain this part of the new method, it will be convenient first to describe what seems to us now to be the best means of supporting the mirrors in contact with the ends of the bars and nearly perpendicular to their axes.

Figure 13 is part of Fig. 1, repeated for convenience. Suppose that h and g are rigorously parallel, and that e' and f' are also rigorously parallel; that is, with an error not more than one second of arc. Suppose, also, that the mirrors and bars are rigidly connected. If, now, we move the bar c' about some point k until f' is parallel to h, e' will also be parallel to g. Let us now move d in the direction of its length, so that f' and h coincide;

[1] Michelson, Travaux et Mémoires du Bureau International, tome 11, p. 137.

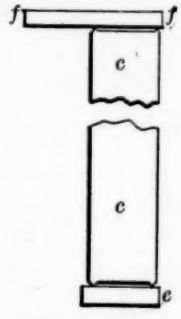
then the difference of length of c' and d will be equal to the perpendicular let fall upon g from *any point* in e'. It is not necessary, with this adjustment of mirrors and bars, to select that perpendicular which coincides with the axes of the two bars, nor to make these two axes coincide.



Figs. 13 and 14.

Now, imagine g replaced by two mirrors g and g', Fig. 14, both parallel to h. It is obvious that we might make the distance between g and h equal to the distance between e' and f' when c' is at zero, and the distance between g' and h equal to the distance between e' and f' when c' is at, say, 100° C.; then the comparison of c' and d at 0° C. would require only the counting of a small number of wave-lengths, the number depending on the care used in securing equality of the distances involved. The same would be true of the comparisons at 100° C. The distance gg' can then be determined, by interferential observations, at leisure, either by direct counting of wave-lengths, or by the elegant method used by Michelson in his interesting and important work "On the Determination of the Value of the Meter in Wave-Lengths."[1]

The use of two mirrors, g and g', involves the measurement of perpendiculars connecting the planes e' and g, e' and g', but not coincident with the axes of the bars, because the mirrors g and g' cannot be superposed. Further, it implies the ability to support all the mirrors e', f', g, g', and h, in positions which shall remain unchanged throughout all the measurements required for a given bar. For this purpose, it is convenient to dispense with adjusting-screws and movable supports for the mirrors.



Figs. 15 and 16.

Figure 15 shows how the mirror e of Fig. 1 would be supported. c, Fig. 15, represents part of the bar, whose end is wrought into three points, on which the mirror e rests. The adjustment of the mirror so as to be perpendicular to the axis of the bar is effected

[1] Travaux et Mémoires du Bureau International, tome 11.

by a careful reduction of the height of one or another of these points, an operation which practice makes comparatively easy. To c is clamped in any convenient way a holder which supports the springs used to apply a constant pressure holding e in contact with the three points. Figure 16 shows one way in which the other mirror ff may be supported. Figure 17 shows the principle utilized in making e and f parallel to each other. The bar is attached to an axis bearing pivots at each end, on which it may be made to rotate. This axis might well be made tubular, so that the bar should be placed within it. If now we use a telescope with a collimating eyepiece, we may make successively the two mirrors perpendicular to the axis of revolution, and so parallel to each other. Instead of the telescope, a certain interferential apparatus may be used, with which a considerably greater accuracy of adjustment can be secured if necessary.

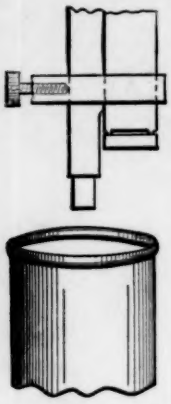
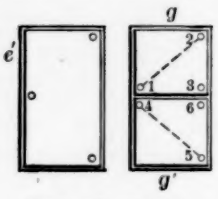


Fig. 17.

When mirrors are thus supported on points which are incapable of slight changes of position, and when care is used to make the pressure of the springs uniform, the constancy of position of the mirrors leaves nothing to be desired. Using supports of this kind, we can adjust the mirrors g and g', Fig. 14, with the necessary constancy. Figure 18 represents the mirror e of Fig. 15, as seen from the front, the end of the bar, and the three supporting points. The mirrors g, g', of Fig. 14, are shown from the front in Fig. 19; g is supported on three points wrought on the upper half of the end of the bar, and g' on three other points on the lower half. One of these mirrors is first made parallel to h, Fig. 14, by the method before explained. Then the second is made parallel to the first by any convenient method. As is seen in the side elevation, Fig. 20, the mirrors are not in the same



Figs. 18 and 19.

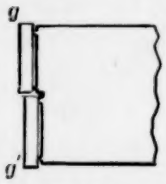


Fig. 20.

plane, and the perpendicular distance between the two reflecting surfaces is to be made approximately equal to the expansion to be measured.

After the mirrors g, g' have been made parallel to the mirror h, Fig. 14, the points **3** and **6**, Fig. 19, are lowered so that the mirrors are made to rotate about lines drawn through **1** and **2**, and through **4** and **5**. The amount of this rotation may be one minute of arc or less, according to the width of interference bands desired by the observer. If these mirrors were left parallel to the mirror h, Fig. 14, the coincidence of one of them with e', Fig. 14, would be shown, in the conditions adopted, by uniform darkness covering the whole face of the mirror, and the counting of wave-lengths would be a counting of alternations of light and darkness. But most observers would prefer to count the march of a series of parallel light and dark bands; hence a suitable inclination is given to g and g'. In order that this inclination shall introduce no error, we have simply to mark the lines joining **1** with **2** and **3** with **4**, and to establish our final coincidence along these lines, and not at some point taken at random.

It may be well to describe the whole course of an observation with this apparatus in the form last described. Let us suppose the bars and mirrors, just before the observation begins, to be placed as in Fig. 21, so that we shall have to move h in the direction of the arrow in order to make h and f' coincide, and that, afterwards, we shall have to continue the motion in the same direction to make g and e' coincide.

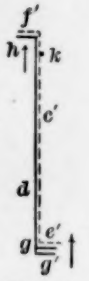


Fig. 21.

By means of the adjustments which move the bar e' in altitude and azimuth about some point k, we give to f' and h a slight inclination whose amount and direction are arbitrary; let us assume that, if f' and h are first made parallel, the observer depresses the larger segment of e' by ten or twenty seconds of arc. With monochromatic light and with the telescope giving distinct vision of the mirrors f' and h, we shall see interference bands as shown at I, Fig. 22. It is by means of these bands that the amount and direction of the inclination of the mirrors is governed. Next, we illuminate part of the surface of f' and h, Fig.

21, with white light; the interference bands of the monochromatic light will be scarcely visible over this part of the field, as shown in *II*, Fig. 22. We now slowly move *h* nearer to *f'*, Fig. 21; in due time, colored interference bands in white light begin to be seen as shown at *III*, Fig. 22, as seen in an inverting telescope. Continuing the motion of *h*, we see the colored bands move upwards and cover the whole surface of the mirrors, and we stop when the central black fringe comes near to the center of the mirror, as at *IV*. We now carefully diminish the inclination of the mirrors, which makes the interference bands increase in width, as at *V*; we keep the central black fringe near the center of the mirrors and diminish the inclination till, finally, we have the mirrors parallel and the black fringe covering the whole surface of the mirrors, as at *VI*. We then move *h* backwards and forwards a tenth of a wave-length, leaving it in the position which gives the maximum of darkness; the planes *f'* and *h* now coincide.

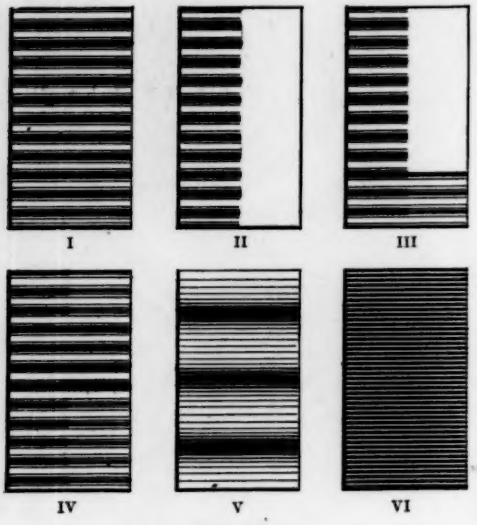


Fig. 22.

We now illuminate the mirrors *e'* and *g*, Fig. 21, with monochromatic light, and direct the telescope upon them, and observe a series of parallel interference bands covering both mirrors, as at *I*, Fig. 23. The line connecting the points 1 and 2, Fig. 19, is indicated in Fig. 23. If some one of the black interference bands exactly coincides with this fiducial line, the distance between the planes *e'* and *g*, Fig. 21, is a whole number of half-wave-lengths. In the figure, this is the case; if not, we should first measure or estimate the fractional part of a wave-length; measurement is easy, but estimation is sufficient. The observer

will have found by trial that, as we bring g nearer e', Fig. 21, the fringes in Fig. 23 will move upwards. The distance from the fiducial line to the fringe which will first cross the line is to be estimated. This fraction is recorded, and is verified after returning to the mirrors $f'h$, and repeating the adjustment for coincidence. Then we introduce white light, as shown at *II*, Fig. 23. We now carefully move g, Fig. 21, nearer e', and count the black bands which pass the fiducial line 1 2, Fig. 23, until we are warned

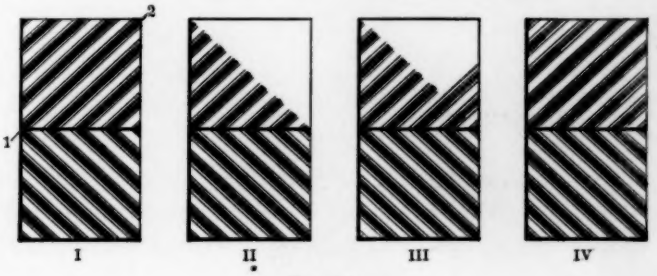


Fig. 23.

by the appearance of colored bands in white light, as at *III*, that we are near the final coincidence. The count is ended when the black band in white light coincides with the fiducial line, as at *IV*. The whole number counted, together with the fraction first estimated, gives the difference in the effective lengths of the two bars; using the phrase "effective length" to mean the perpendicular distance between the fiducial line and the surface of the remote mirror. This length is made up, when the mirrors are supported as now described, of a length of metal together with a thickness of glass.

If we now change the temperature of the bar c, Fig. 1, by a suitable amount, we may repeat the observation, using only the mirror g', Fig. 20, in place of g. The lengths and temperatures may be so chosen that the counting occupies only a minute, and may therefore be repeated as often, and throughout as long a time, as may be thought proper. Afterwards we have to determine the difference of the lengths gh and $g'h$, either by direct count or by Michelson's method; in either way, the determination is easy and accurate.

When we wish to repeat the pair of observations with a different interval of temperature, the distance between the planes g and g' will need to be changed accordingly. But we need not repeat the adjustment of the points of support, Fig. 19. Instead of this, we provide ourselves with a number of plane parallel glass plates of thicknesses varying by a tenth or a twentieth of a millimeter, and we replace the plate g', Fig. 20, by one whose thickness is greater or less by the desired amount.

It is also possible to mount the mirrors gg', Fig. 14, upon sliding carriers, adjust them to the desired position, and clamp them. In this case, there would be left something to be desired as to the fixity of the effective length gh, $g'h$, which would require attention.

If observations by our new method were made in a good vacuum, and if the temperature of one bar were quite constant, the differences of the observed numbers of wave-lengths would represent the real changes of length of the bar whose temperature is made to vary. Neither of these conditions being always obtained, two numerical corrections must be applied to the observed numbers.

(To be concluded.)

THE VISCOSITY OF POLARIZED DIELECTRICS.

By A. Wilmer Duff.

THE list of changes observed in the mechanical properties of a non-conducting liquid on subjecting it to electrostatic stress is a very short one, and that notwithstanding the fact that the subject is one of the greatest importance, because of its bearing on the question of the nature of electrification and electrostatic stress.

Quincke[1] found that some liquid dielectrics expanded in volume and others contracted under electrostatic stress; but Röntgen,[2] on repeating the experiments, failed to find the contractions and explained the expansions as due to heat produced by the passage of electricity through the dielectric; hence no satisfactory conclusion has been reached. Kerr[3] discovered that a liquid dielectric became double refracting under electrostatic stress, some liquids acting "like glass extended in a direction parallel to the lines of electric force" and others "like glass compressed in a direction parallel to the lines of electric force." These phenomena of Kerr are, however, only inferentially mechanical. W. König,[4] Pagliani,[5] Noack,[6] examined the viscosity of the dielectric, but failed to find any alteration of it due to electrostatic stress. Faraday's observation[7] that fibers of silk in the polarized dielectric stretched themselves in the direction of the electric force is an illustration of the effect of different inductive capacities. Hence, it would seem that we still lack positive knowledge as to the effect of electrostatic stress on the mechanical properties (density, compressibility, viscosity, etc.) of liquid dielectrics.

[1] Wiedemann's Annalen, X., p. 521, 1880.
[2] Wiede. Ann., XI., p. 780, 1880.
[3] Philosophical Magazine, Nov., Dec., 1875; Aug., Sept., 1879.
[4] Wiede. Ann., XXV., p. 618; 1885.
[5] Ac. Torino, 20, p. 615, 1885; 22, p. 1, 1887 (Winkelmann's Physik).
[6] Wiede. Ann., XXVII., p. 289, 1886 (Winkelmann's Physik).
[7] Experimental Researches, XII., 1350.

About a year ago I undertook to reëxamine the question of a possible viscosity variation, but at first failed to find evidence (at least conclusive evidence) of any such effect. The method employed was not that of measuring the flow through a capillary tube placed between condenser plates (König), but one depending on the rate of descent of small spheres through a viscous liquid. Failing to find the effect sought, I have lately repeated the experiments with more care, rather with a view to fixing a superior limit to a possible viscosity variation than with the hope of actually discovering such a variation. The results obtained seem, however, to point clearly to the existence of the variation in question. Not many liquids have as yet been examined, as it seemed of more importance at first to thoroughly test one liquid under varying conditions. The results given in the present paper will, therefore, be limited to two or three liquids; and the extension of the methods employed to other liquids, together with a study of the law of variation of the effect under varying intensity of stress, will be left to a future paper.

The capillary tube method seemed to me, at first, objectionable, because of the difficulty of maintaining a sufficiently constant temperature for the requisite length of time; for, as is well known, the viscosity of a liquid varies very greatly with its temperature (as much as 10 per cent per degree in glycerin), and even a small change of temperature might completely mask other considerable changes of viscosity. This will appear from the following figures given by König, carbon bisulphide being the liquid used.

Temperature.	Viscosity.		Temperature.	Viscosity.	
17.5	0.00413	Plates uncharged.	17.8	0.00422	Plates uncharged.
16.8	0.00447	Plates uncharged.	17.8	0.00420	Plates charged.
17.6	0.00418	Plates charged.	16.2	0.00410	Plates uncharged.
18.6	0.00411	Plates uncharged.	16.1	0.00414	Plates charged.

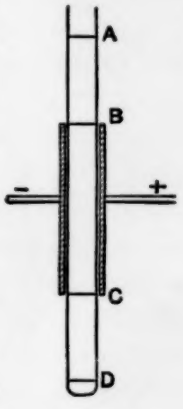
No doubt a much greater constancy of temperature than the above might be readily attained, but there seemed to be more hope in employing some method that would eliminate the effect of temperature variations.

The method adopted was to make use of the formula[1] found by Stokes for the rate of descent of a sphere through a viscous fluid. The steady velocity attained by a sphere falling through a fluid is:

$$V=\frac{2}{9}\frac{\rho-\sigma}{\eta}ga^2,$$

ρ being the density of the sphere, σ that of the fluid, a the radius of the sphere, and η the viscosity of the fluid. This is on the assumption that there is no finite slipping between the sphere and the liquid and that none such exists (at least in the case of such materials as are used in what follows) has been shown by Mr. O. G. Jones.[2] In the remainder of this paper the only assumption made from the above formula is that as the viscosity of a liquid increases, the time of descent of a sphere through a given distance in the medium also increases. (The possibility of a sufficient change in the density of the liquids used in my experiments or of an expansion of the glass tank employed will be adverted to later.)

If, then, a tube be filled with a viscous liquid and the time of descent of a small drop of mercury (the use of which will be justified later) through the length of the tube be noted and the tube be then inverted between the plates of a charged condenser, any change of viscosity would be shown by a change in the time of descent of the mercury drop. This would imply, of course, a constant temperature for the liquid, but such being unattainable, the following modification was made for approximately eliminating the temperature effect. Instead of the whole tube being placed between the condenser plates, the plates were placed opposite the middle half of the tube only, rings being etched around the tube to mark off the upper quarter, *AB*, the middle half, *BC*, and the lower quarter, *CD*, of the length through which the descent was observed. By means of two carefully tested stop watches, the ratio of the time of descent through *BC* to the sum of the



[1] Lamb's Motion of Fluids (1879), § 184.

[2] Proceedings Physical Society of London, 1894, or Nature, Feb. 22, 1894.

times of descent through AB and CD was obtained. This ratio would, of course, be independent of the actual size of the drop (providing its steady velocity was attained before reaching A), so that the inversion of the tube was unnecessary. Another drop was then allowed to descend, and the above ratio again obtained, the condenser being in this case charged. By this method of taking ratios, we get rid of the effect of any steady variation in temperature affecting the whole tube equally. Hence, barring local inequalities in temperature variation, if any change of the viscosity of the liquid between B and C were produced by the electrostatic stress, we should find a difference between the ratios obtained above.

Using the above method, I made seven series of observations on glycerin, which, though a very imperfect insulator, was chosen because of its suitable viscosity. The work was carried on in a room not artificially heated. Of the seven series of observations taken, two seemed to point clearly to the existence of the variation sought, three others showed one break each in the consistency of their indications, and the others were quite irregular, though giving a mean result of the same sign as before. These observations, therefore, were, in the absence of any previous knowledge as to such a variation, of little value in themselves; but, as they have become of some importance when taken in connection with later results by a more satisfactory method, I give the two following series as samples, the first being fairly representative of the consistent sets obtained, and the second of the semi-consistent sets. The condenser was kept charged by means of a small influence machine worked with a fair degree of uniformity. The distance between the condenser plates was 3.5 cm. As a rough indication of the potential I give the mean length of spark between the terminals of the influence machine.

Supposing for the moment that those results which were consistent among themselves were not misleading in their indications, the most probable source of the inconsistency of the others was irregular temperature fluctuations. Hence it seemed desirable to continue the observations under the following more favorable conditions: (1) in a room of more constant temperature, (2) with

Series I.

GLYCERIN.

Condenser.	T_1 = sum of times of descent through AB and CD.	T_2 = time of descent through condenser.	T_2/T_1.	Excess of T_2/T_1 when condenser was charged.	Temperature.	Remarks.
	min. sec.	min. sec.				
Uncharged.	4 15.60	4 12.40	0.9875	+0.0074	19.8	
Charged. .	4 7.75	4 6.50	0.9949			
Uncharged.	4 28.25	4 26.50	0.9935	+0.0044	19.8	
Charged. .	4 4.75	4 4.25	0.9979			
Uncharged.	5 1.75	4 54.75	0.9768	+0.0116	—	
Charged. .	3 36.50	3 34.00	0.9884			Interval.
Uncharged.	5 4.25	4 54.25	0.9761	+0.0030	20.9	
Charged. .	4 36.00	4 30.25	0.9791			
Uncharged.	3 6.25	3 2.50	0.9799	+0.0076	21.0	
Charged. .	4 0.50	3 57.50	0.9875			
Uncharged.	3 31.20	3 28.00	0.9846	+0.0048	21.0	
Charged. .	3 9.50	3 7.50	0.9894			
Uncharged.	4 6.20	4 1.50	0.9797	+0.0169	21.2	
Charged. .	3 53.00	3 52.20	0.9966			
Uncharged.	3 13.70	3 10.20	0.9819	+0.0083	—	
Charged. .	4 1.50	3 59.20	0.9902			

Mean length of spark, 1.6 cm.

a liquid less sensitive to temperature, but still of a suitable viscosity, (3) with the use of a large body of liquid whose temperature might not change so readily. I therefore made a large tank 60 cm. × 30 cm. × 2 cm. (internal dimensions). This was constructed of sheets of plate glass, solidly clamped on an open rectangle of hard wood, with rubber tubing between the glass and the wood. The tank was marked off into an upper quarter, a middle half, and a lower quarter, as in the case of the tube. Sheets of tin-foil glued to the middle halves of the plates served as a condenser, the edges of the tin-foil being thickly coated with paraffin wax. The observations were thenceforward made in a cellar of the Purdue Electrical Building. To protect the tank

SERIES II.

GLYCERIN.

Condenser.	T_1 = sum of times of descent through AB and CD.	T_2 = time of descent through condenser.	T_2/T_1.	Excess of T_2/T_1 when condenser was charged.	Remarks.
	min. sec.	min. sec.			
Uncharged . . .	1 7.2	1 7.7	1.007	−0.003	
Charged	1 17.2	1 17.5	1.004		
Uncharged . . .	1 36.0	1 36.0	1.000	+0.005	
Charged	1 37.5	1 38.0	1.005		
Uncharged . . .	1 49.0	1 49.5	1.005	+0.007	
Charged	3 11.2	3 13.5	1.012		
Uncharged . . .	1 38.2	1 39.2	1.010	+0.003	
Charged	1 17.2	1 18.2	1.013		
Uncharged . . .	1 31.0	1 32.0	1.011	+0.002	
Charged	1 42.2	1 43.5	1.013		
Uncharged . . .	1 21.7	1 21.7	1.000	+0.003	
Charged	0 58.2	0 58.5	1.003		

Mean length of spark, 1.3 cm.

from air currents, it was surrounded by glass screens. A bright background for the observation of the descending drops was obtained by reflecting sky light from tilted glass plates. The most suitable time for working was found to be between one and four o'clock in the afternoon, during which period the temperature of the cellar usually reached a maximum, and varied but slowly.

The results obtained were at first somewhat irregular, until the following simple modification was made. The drops were started in regular rapid succession, and in calculating, the ratio T_2/T_1, obtained with the condenser uncharged, was compared with the mean of the ratios obtained immediately before and immediately after with the condenser charged. This process nearly eliminated any uniformly progressive temperature variation from above or below. It also eliminated any steady variation in the rates of the stop watches used for taking time. Only days of normal tempera-

ture conditions were chosen for work. These conditions being observed, the observations showed thenceforward an almost invariable consistency.

The following series of readings seem to show that castor oil suffers an increase of viscosity under electrostatic stress.

Series III.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	2 37.75	2 31.0	0.9572	—	—	—
Uncharged .	3 40.50	3 30.0	—	0.9564	0.9524	+0.0040
Charged . .	3 40.25	3 30.5	0.9557	—	—	—
Uncharged .	3 7.75	2 58.5	—	0.9546	0.9507	+0.0039
Charged . .	3 18.75	3 9.5	0.9535	—	—	—
Uncharged .	4 14.75	4 2.0	—	0.9534	0.9500	+0.0034
Charged . .	3 39.75	3 29.5	0.9534	—	—	—
Uncharged .	3 21.75	3 11.5	—	0.9536	0.9492	+0.0044
Charged . .	3 20.25	3 11.0	0.9538	—	—	—
Uncharged .	2 58.75	2 49.5	—	0.9537	0.9482	+0.0055
Charged . .	2 31.00	2 24.0	0.9536	—	—	—
Mean					0.9501	+0.0043

Mean increase	0.45 per cent.
Mean spark length	3 cm.
Temperature of oil decreased from	17°.5 to 16°.8
Temperature of air decreased from	17°.5 " 17°.0

SERIES IV.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	1 19.25	1 16.25	0.9621	—	—	—
Uncharged .	2 8.00	2 3.00	—	0.9583	0.9609	−0.0026
Charged . .	1 50.00	1 45.00	0.9545	—	—	—
Uncharged .	2 42.00	2 35.00	—	0.9577	0.9568	+0.0009
Charged . .	1 55.00	1 50.50	0.9609	—	—	—
Uncharged .	1 52.50	1 47.00	—	0.9638	0.9511	+0.0027
Charged . .	1 56.00	1 52.25	0.9668	—	—	—
Uncharged .	2 27.25	2 21.25	—	0.9641	0.9593	+0.0048
Charged . .	2 36.00	2 30.00	0.9615	—	—	—
Uncharged .	2 30.00	2 23.00	—	0.9669	0.9533	+0.0136
Charged . .	3 0.40	2 55.40	0.9723	—	—	—
Uncharged .	2 39.25	2 33.75	—	0.9702	0.9655	+0.0047
Charged . .	2 44.75	2 39.50	0.9681	—	—	—
Uncharged .	2 42.75	2 36.75	—	0.9670	0.9631	+0.0039
Charged . .	2 34.00	2 28.75	0.9659	—	—	—
Uncharged .	2 11.25	2 6.00	—	0.9650	0.9600	+0.0050
Charged . .	1 58.50	1 54.25	0.9641	—	—	—
Uncharged .	1 45.90	2 36.75	—	0.9631	0.9595	+0.0036
Charged . .	2 5.75	2 1.00	0.9622	—	—	—
Uncharged .	2 5.75	2 0.50	—	0.9619	0.9583	+0.0036
Charged . .	2 23.75	2 18.25	0.9617	—	—	—
Mean					0.9587	+0.0040

Mean increase . . . . . 0.42 per cent.
Temperature of air increased from . . . . . 13° to 13°.5
Temperature of oil . . . . . not taken; light bad.
Mean spark length . . . . . 3.6 cm.

Series V.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	3 7.75	2 58.75	0.9521	—	—	—
Uncharged .	2 19.00	2 12.50	—	0.9598	0.9532	+0.0066
Charged . .	3 20.00	3 13.50	0.9675	—	—	—
Uncharged .	2 36.00	2 30.00	—	0.9636	0.9616	+0.0020
Charged . .	4 1.75	3 52.00	0.9597	—	—	—
Uncharged .	2 31.75	2 25.50	—	0.9612	0.9588	+0.0024
Charged . .	2 21.00	2 15.75	0.9628	—	—	—
Uncharged .	3 20.75	3 11.00	—	0.9640	0.9515	+0.0125
Charged . .	2 30.75	2 25.20	0.9652	—	—	—
Uncharged .	2 39.75	2 33.25	—	0.9613	0.9593	+0.0020
Charged .	3 8.25	3 0.25	0.9575	—	—	—
Mean					0.9568	+0.0051

Mean increase . . . . . 0.53 per cent.
Mean spark length . . . . . 3.2 cm.

In these results it should be noted how extremely good the agreement between the differences is when the mean ratios in the fifth column are changing steadily and not irregularly. Series III. and the latter half of IV. show this especially well. The ratio .9545 (column 4, Series IV.) is so very different from the other ratios in the same column that it must be due to a blunder in taking time. This will account for the only negative difference obtained.

Two other series of readings were made, but were marked "rejected" after calculating the ratios obtained with the condenser uncharged and before calculating those obtained with the condenser charged. This rejection was made on the ground that the ratios obtained with the condenser uncharged varied irregularly among themselves, and hence would be unlikely to give consistent indications. These were all the readings so made.

I then proceeded to inquire into other possible explanations of this apparent increase of viscosity.

(1) Temperature. It is well known that heat is produced by rapidly charging and discharging a condenser. Hence an increase of temperature of the liquid might be produced by fluctuations of the potential to which the condenser was charged. This increase would, however, be very slight, owing to the mass of liquid employed, and to the fact that the influence machine was worked pretty uniformly. Very accurate readings of the temperature of the liquid were not possible in the cellar where I worked, for these would bring the body close to the tank, and thus produce a local heating of the liquid. Rough readings were, however, made, and showed no uniform variations. Any changes of temperature produced in the liquid itself by variations in the dielectric stress acting on it may be rejected at once as a sufficient cause for the phenomenon, for they would produce a decrease of viscosity. Variations in the temperature of the liquid, due to heat produced in the glass, might conceivably produce the effect observed; for, owing to the low conducting power of the liquid, the maximum of the temperature increase produced in the center of the liquid by heat transmitted from the glass might be postponed until the time of descent of the next drop. A glance, however, at the preceding results will show that the time of descent varied so widely that such a fluctuation could not be of a sufficiently periodic nature to account for the results. There were, moreover, frequent short interruptions of work.

An examination of Stokes' formula will show that an alteration of the difference of density of mercury and castor oil due to heating would affect the time of descent; but a short calculation will show that the requisite temperature change would be enormous.

Moreover, the radius of the mercury drop is contained in the formula; but a sufficient alteration of it would require at least 25° change of temperature.

A thermal expansion of the middle halves of the glass plates with a consequent alteration of the apparent time of descent may also be dismissed as insufficient.

(2) The drops might be deflected horizontally by the force of the field. This seemed very improbable, but to test it a small millimeter scale was placed at the bottom of the tank and the distances of the landing places of the drops from the center of the tank noticed. The following were the results:

Plates charged	$2\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	2
Plates uncharged	$2\frac{1}{2}$	2	2	

Hence there was no appreciable horizontal deflection.

(3) The shaking of the table when the machine was being worked to charge the condenser, might possibly affect the velocity of the drops. (The shaking, however, was slight, and the table a heavy one.) Hence, a series of observations were taken with the machine in action all the time during the descent of the alternate drops. The results (all positive) showed that the effect still remained when this possible cause was eliminated. As a further precaution, the rates of the stop watches were compared and found unaffected by the action of the machine. These observations need not be quoted, as the matter is sufficiently settled by the results obtained later with heavy paraffin oil.

(4) The effect might be due to a retardation of the drop where it left the field of the condenser. This seemed also a very improbable explanation, but to guard against it the condenser was discharged before the drop reappeared, the time of reappearance being estimated from the time of descent through the upper quarter of the tank.

(5) The drops of mercury, being conductors, might be deformed by the action of the field. If they became prolate along the lines of force, the time of descent would probably be increased. As this seemed a not unlikely source of fallacy, a series of observations were made with small shot instead of mercury drops. Care was taken to get shot as nearly perfectly spherical as possible, the sphericity being tested by examination with a magnifying glass, and by observing the degree of smoothness with which the shot rolled on a glass plate. The following series of readings seems to dispose of this source of suspicion. It will be noticed

that it gives nearly the same percentage alteration of viscosity as did the mercury drops.

SERIES VI.

CASTOR OIL.

(*Tested by Descent of Small Shot.*)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	2 15.00	2 7.50	0.9444	—	—	—
Uncharged .	1 44.40	1 38.75	—	0.9461	0.9459	+0.0002
Charged . .	1 36.40	1 28.25	0.9479	—	—	—
Uncharged .	2 24.75	2 17.00	—	0.9476	0.9465	+0.0011
Charged . .	2 8.25	2 1.50	0.9474	—	—	—
Uncharged .	2 39.75	2 31.50	—	0.9509	0.9484	+0.0025
Charged . .	2 22.50	2 16.00	0.9544	—	—	—
Uncharged .	2 4.25	1 57.75	—	0.9526	0.9477	+0.0049
Charged . .	3 18.75	3 9.00	0.9509	—	—	—
Uncharged .	2 41.75	2 33.25	—	0.9492	0.9474	+0.0018
Charged . .	2 23.00	2 15.50	0.9476	—	—	—
Uncharged .	2 16.25	2 8.75	—	0.9520	0.9450	+0.0070
Charged . .	2 17.75	2 11.75	0.9564	—	—	—
Uncharged .	2 15.75	2 8.25	—	0.9523	0.9448	+0.0075
Charged . .	2 20.00	2 12.75	0.9482	—	—	—
Mean					0.9465	+0.0036

Mean increase	0.38 per cent.
Mean spark length	2.5 cm.
Temperature of oil increased from	9° to 9°.2
Temperature of air decreased from	10° " 8°.8

(6) The shot, being not perfectly spherical, might, when within the condenser, take an average "set" with the greatest diameter parallel to the lines of force. That this cause could be sufficient seems improbable, the shot being so nearly perfectly spherical. Moreover, it seems very improbable that this and the deformation of the mercury drops should produce so nearly the same percentage change of velocity. If such a "set" of the shot occurred

within the condenser some trace of this would be likely to persist during the remainder of the descent, since the shot ordinarily rotated only very slowly in descending. A lengthy series of readings of the times of descent through the highest and the lowest quarter of the tank failed to show any such effect. A conclusive test would be to replace the shot by spheres of non-conducting material of about the same S.I.C. as the dielectric. To this end many kinds of beads, pills, and seeds were tested, but none were found sufficiently spherical and of suitable density. Finally, some carefully selected imitation pearls (so-called "Roman" pearls), made of hard wax were tried. They gave the following series of results, which seems to finally settle the question.

SERIES VII.

CASTOR OIL.

(Tested by Descent of Imitation Pearls.)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser uncharged.	Mean of successive values of T_2/T_1; condenser uncharged.	T_2/T_1; condenser charged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Uncharged .	2 59.00	2 51.75	0.9595	—	—	—
Charged . .	2 35.25	2 29.25	—	0.9577	0.9613	+0.0036
Uncharged .	2 10.25	2 4.50	0.9559	—	—	—
Charged . .	1 58.25	1 53.25	—	0.9554	0.9577	+0.0023
Uncharged .	3 14.00	3 5.25	0.9549	—	—	—
Charged . .	2 20.00	2 13.75	—	0.9547	0.9554	+0.0007
Uncharged .	3 1.25	2 53.00	0.9545	—	—	—
Charged . .	2 33.75	2 28.50	—	0.9606	0.9659	+0.0053
Uncharged .	2 0.50	1 56.25	0.9667	—	—	—
Charged . .	3 5.10	2 58.75	—	0.9621	0.9657	+0.0036
Uncharged .	2 13.25	2 7.75	0.9576	—	—	—
Charged . .	2 59.50	2 51.50	—	0.9549	0.9554	+0.0005
Uncharged .	2 37.75	2 29.75	0.9523	—	—	—
Mean					0.9559	0.0027

Mean increase . . . . . 0.28 per cent.
Mean spark length . . . . . 2.6 cm.
Temperature of air increasing from . . . . . 8°.75 to 9°.0
Temperature of oil increasing from . . . . . 8°.65 " 8°.8

These results are, as might be expected, less regular than preceding ones. Omitting from the column of differences the numbers which vary widely from the mean, the average percentage increase of viscosity is roughly the same as before. This agreement is a further argument against the existence of finite slipping between the liquid and the descending sphere.

From all the preceding we seem justified in concluding that the viscosity of castor oil is increased by about one half of one per cent when the oil is subjected to the electrostatic stress produced by a potential gradient of about 27,000 volts per centimeter or 90 E.S., C.G.S. units of potential per centimeter.

With reference to the preceding statement, two remarks may be made; first, that the electrostatic field was only steady to the

SERIES VIII.

HEAVY PARAFFIN OIL.

(*Tested by Descent of Mercury Drops.*)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser uncharged.	Mean of successive values of T_2/T_1; condenser uncharged.	T_2/T_1; condenser charged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Uncharged.	3 26.6	3 27.8	1.0058	—	—	—
Charged. .	2 40.3	2 42.0	—	1.0069	1.0075	+0.0006
Uncharged.	3 17.2	3 18.8	1.0081	—	—	—
Charged. .	2 7.9	2 9.2	—	1.0087	1.0102	+0.0015
Uncharged.	3 45.6	3 47.7	1.0093	—	—	—
Charged. .	2 47.4	2 48.0	—	1.0084	1.0036	−0.0048
Uncharged.	2 14.0	2 15.0	1.0075	—	—	—
Charged. .	1 51.5	1 52.4	—	1.0062	1.0081	+0.0019
Uncharged.	2 41.8	2 42.6	1.0049	—	—	—
Charged. .	2 20.8	2 21.4	—	1.0067	1.0043	−0.0024
Uncharged.	2 56.8	2 57.8	1.0086	—	—	—
Charged. .	2 7.8	2 8.4	—	1.0056	1.0047	−0.0009
Uncharged.	2 30.4	2 30.8	1.0027	—	—	—
Mean					—	−0.0008

Mean spark length . . . . . 3 cm.
Temperature of oil decreasing from . . . . . 5°.1 to 4°

extent that the influence machine was worked fairly regularly; second, that the estimate of the field strength, being made from the mean sparking distance and the equation deduced by Chrystal

Series IX.

HEAVY PARAFFIN OIL.

(Tested by Descent of Mercury Drops.)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser uncharged.	Mean of successive values of T_2/T_1; condenser uncharged.	T_2/T_1; condenser charged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Uncharged .	4 49.4	4 49.0	0.9986	—	—	—
Charged . .	5 4.1	5 4.3	—	0.9993	1.0007	+0.0014
Uncharged .	4 56.0	4 56.0	1.0000	—	—	—
Charged . .	4 33.0	4 32.2	—	0.9986	0.9971	−0.0015
Uncharged .	3 30.8	3 30.2	0.9972	—	—	—
Charged . .	4 14.0	4 14.4	—	1.0002	1.0016	+0.0014
Uncharged .	5 4.0	5 5.0	1.0033	—	—	—
Charged . .	3 37.6	3 37.8	—	1.0040	1.0009	−0.0031
Uncharged .	4 33.7	4 35.0	1.0047	—	—	—
Charged . .	4 0.8	4 0.8	—	1.0028	1.0000	−0.0028
Uncharged .	3 45.0	3 45.2	1.0009	—	—	—
Charged . .	4 30.8	4 31.0	—	1.0004	1.0007	+0.0003
Uncharged .	3 48.0	3 48.0	1.0000	—	—	—
Charged . .	3 25.8	3 25.6	—	1.0015	0.9990	−0.0025
Uncharged .	3 14.4	3 15.0	1.0031	—	—	—
Charged . .	4 32.4	4 32.2	—	0.9981	0.9993	+0.0012
Uncharged .	3 51.6	3 50.0	0.9931	—	—	—
Mean					—	−0.0007

Mean spark length . . . . . 3.5 cm.
Temperature of air increasing from . . . . . 7°.3 to 7°.50
Temperature of oil decreasing from . . . . . 8°.2 " 7°.75

to represent Baille's results for the potential necessary to a given spark length, is only the roughest approximation.

It having now been shown pretty conclusively that at least one dielectric undergoes a change of viscosity under dielectric stress, it becomes an *a priori* probability that other dielectrics

are similarly affected. Hence there is no longer any cause to doubt that the earlier experiments on glycerin (Series I. and II.) were reliable in their indications, and that glycerin also has an increased viscosity under electrostatic stress.

The only other liquid so far tested by the present method is heavy paraffin oil (specific gravity = .883). As will be seen by the series of readings VIII. and IX., the evidence as to this liquid is hardly conclusive. Mercury drops were used in this case.

From the above results it seems possible that heavy paraffin oil suffers a decrease of viscosity, the decrease being, however, much less than the increase in the preceding liquids, and much more difficult to determine. It should be noted that the failure to find an increase of viscosity in paraffin oil, or, still more, the discovery of a decrease is a strong evidence of the validity of the method employed in these experiments. The only detraction from this argument is that much smaller drops of mercury were used in the case of paraffin oil than in the case of castor oil. In all other particulars the method was the same.

The method described in this paper can only be applied to very viscous liquids. For testing less viscous liquids I have constructed two separate forms of apparatus, in which capillary tubes are used in such a way as to eliminate temperature effects. These have already been applied to testing several liquids, and to examining the effect of different intensities of dielectric stress and different degrees of unsteadiness or non-uniformity of field. These results will be given in another paper.

PHYSICAL LABORATORY, PURDUE UNIVERSITY,
March, 1896.

NOTE ON THE THEOREM OF CLAUSIUS.

By Edgar Buckingham.

The treatments commonly given of the expression $\int \frac{dQ}{T}$, and the proofs of the existence of a function η, the entropy, such that

$$d\eta = \frac{dQ}{T} \tag{1}$$

seem to me to be very incomplete. Even if we confine ourselves to the consideration of reversible processes, we have difficulty in finding any book where the subject is treated in such a way as to be clear to the ordinary student.

One small difficulty is that most writers only prove that

$$d\eta = \frac{dQ}{T}$$

when the work, w, done on the system by the outside forces is purely mechanical; in fact, when it is of the form

$$dw = -p\,dv, \tag{2}$$

though they afterward apply the theory to problems where electrical and other non-mechanical forms of energy enter. A greater fault is the failure to consider at all systems the state of which is defined by more than two independent variables. Such systems have, indeed, been considered, but work like that of Voigt[1] and Duhem,[2] who assume familiarity with the language at least of geometry of more than three dimensions, is not suitable for the use of most students of physics. If we are to use entropy, free energy, or the thermodynamic potential, we must, to satisfy ourselves, make sure that the entropy and its derivatives are definite functions before we go farther. I have felt, in teaching, the need of a simple but careful treatment of the subject, and as I have not

[1] Kompendium der Theoretischen Physik, Vol. I., Part III., chap. I., § 2.

[2] Journal de Mathematiques, 8, p. 269, and 9, p. 293, 1892 and 1893.

found one, I offer the following remarks mainly in the hope that they may elicit something better on the same theme.

§ 1.

Let our system have throughout the temperature T measured on Thomson's absolute scale. Let its state be completely determined by n independently variable parameters $(c_1 c_2 \cdots c_n)$ in addition to T. Let these parameters be so chosen that a variation of T alone involves no external work. This limitation of the generality of our treatment is practically unimportant, for we always do find it convenient to choose our parameters in this way. In any infinitesimal change involving external work we shall have

$$dw = \frac{\delta w}{\delta c_1} dc_1 + \frac{\delta w}{\delta c_2} dc_2 \cdots + \frac{\delta w}{\delta c_n} dc_n. \tag{3}$$

If we let

$$i_1 = \frac{\delta w}{\delta c_1}, \quad i_2 = \frac{\delta w}{\delta c_2}, \quad i_n = \frac{\delta w}{\delta c_n}, \tag{4}$$

whence (3) reduces to

$$dw = \sum_1^n i_k dc_k, \tag{5}$$

we may call i_k the *force* corresponding to the ***parameter*** c_k, or the *intensity* corresponding to the *capacity* c_k. The product $(i_k c_k)$ is obviously a quantity of the dimensions of energy.

If we assume, as is done, that it is impossible by any cyclic process or processes to obtain an infinite quantity of work from a system which at the end has exactly the same state as at the beginning, the internal energy, ϵ, is determined by the state of the system, or

$$\epsilon = f(T c_1 c_2 \cdots c_n) \tag{6}$$

so that $\frac{\delta \epsilon}{\delta T}$, $\frac{\delta \epsilon}{\delta c_1}$, $\frac{\delta \epsilon}{\delta c_2} \cdots \frac{\delta \epsilon}{\delta c_n}$ are definite functions of $T c_1 c_2 \cdots c_n$.

§ 2.

In the case of Carnot's engine $n = 1$, $c = v$, and $-i = p$, so that we have

$$dw = -p dv \,[1]$$

[1] We may take i and c in any way we please so long as $idc = -pdv$.

For this engine it is easily shown that for any reversible cycle

$$\int \frac{dQ}{T} = 0,$$

whence it at once follows that there exists an entropy function, such that

$$dQ = T d\eta.$$

Since we have the state of the system completely given by two variables, and since

$$p = f(T, v),$$

we may just as well consider p and v as the independent variables and let

$$T = \phi(p, v).$$

We can then represent any process by the movements of an indicator point on a plane diagram with coördinates p and v. An area on the diagram is of the form $\int p dv$ and represents a quantity of work.

§ 3.

Suppose, now, as a first step in advance, that our system is one which does work of *any* sort, but that its state may still be determined by one variable c in addition to the temperature. I have said work of *any* sort, but this must be limited to those forms of energy which are of the same *value* as mechanical energy; *i.e.* forms which are completely convertible, in both directions, with mechanical work.[1]

Among these forms are electrostatic and magnetostatic energy.[2] I shall exclude from consideration all forms of energy, except heat, which do not satisfy this condition. Such a system cannot be coupled directly to a Carnot engine. But if we make the connection by means of a mechanism which is capable of performing this complete mutual transformation of the two forms of energy, Carnot's theorem may be extended at once, and we may say that the

[1] See F. Wald, Die Energie und ihre Entwertung, Leipzig, W. Englemann, 1889.

[2] The question of the applicability of Carnot's theorem to problems involving radiant energy, electrokinetic energy, etc., involves the preliminary investigation of the question whether these forms are completely convertible into mechanical energy, and *vice versa*.

efficiency of *any* reversible engine of which the condition is fixed by one variable in addition to the temperature is equal to the efficiency of Carnot's engine. Hence we have at once the extension of the theorem of Clausius to all such engines or systems, and for any reversible cycle

$$\int \frac{dQ}{T} = 0.$$

Therefore we have

$$dQ = T d\eta,$$

and η is a definite quantity.

The graphic representation of changes in the state of such a system is analogous to that for Carnot's engine. We might use T and c as our coördinates. We might use any two variables, x and y, such that

$$\left.\begin{aligned} x &= f_1(T, c) \\ y &= f_2(T, c) \end{aligned}\right\}, \tag{7}$$

for they fix completely the state of the system, because equations (7) involve the relations

$$\left.\begin{aligned} T &= \phi_1(x, y) \\ c &= \phi_2(x, y) \end{aligned}\right\}, \tag{8}$$

and T and c determine the state of the system. But it is clearest to use as our coördinates c and i, in which case an area on the diagram represents a quantity of work. We may then, if we have sufficient data, draw isothermal and adiabatic lines just as we do on the pv diagram.

A simple example of such a system is a mass of liquid, the condition of which is defined by its temperature, T, and its surface area, S. Here the intensity, i, is evidently the surface tension, σ, and we have

$$\left.\begin{aligned} c &= S \\ i &= \sigma \end{aligned}\right\}. \tag{9}$$

In this particular case the equation $i = f(T, c)$ reduces to $\sigma = f(T)$, since we know that σ is independent of S. The isothermal lines are horizontal if, as usual, we take our force or intensity along the vertical axis. To construct the adiabatic lines we must know the latent heat of isothermal extension, the heat capacity of the system, and $\frac{\delta\sigma}{\delta T}$

Another somewhat similar case is that of the reversible galvanic cell. Here we may most simply take as our parameter, c, the quantity of electricity, q, which has passed through the cell since the time when it was in some standard state. The intensity, i, is then the electromotive force, E. Here again, if the cell is a constant one, $E = f(T)$, and the isothermals are horizontal. To draw the adiabatics we must know the heat of reaction, the heat capacity of the cell, and $\frac{\delta E}{\delta T}$.

§ 4.

Let us now pass on to the general case of a system the state of which is defined by the $(n+1)$ variables

$$Tc_1c_2\cdots c_n.$$

In order to represent the state of the system and its changes by the position and motions of one indicator point we must let the point move in space of $(n+1)$ dimensions, so that as soon as $n > 2$ we are left stranded. But we can make the process somewhat more clear by a graphic representation consisting of n plane diagrams. In the equation for the work of the external forces,

$$dw = i_1dc_1 + i_2dc_2 + \cdots i_ndc_n, \tag{5}$$

where

$$i_1 = \frac{\delta w}{\delta c_1}, \quad i_2 = \frac{\delta w}{\delta c_2}\cdots i_n = \frac{\delta w}{\delta c_n},$$

the forces, i, which keep the system in a state of equilibrium, must depend on the state of the system. There must therefore exist n relations of the form

$$\left.\begin{aligned} i_1 &= f_1(Tc_1c_2\cdots c_n)\\ i_2 &= f_2(Tc_1c_2\cdots c_n)\\ &\cdots\cdots\\ i_n &= f_n(Tc_1c_2\cdots c_n)\end{aligned}\right\}, \tag{10}$$

and since these may be put in the form

$$\left.\begin{aligned} T &= \phi_1(i_1c_1c_2\cdots c_n)\\ T &= \phi_2(i_2c_1c_2\cdots c_n)\\ &\cdots\cdots\\ T &= \phi_n(i_nc_1c_2\cdots c_n)\end{aligned}\right\} \tag{11}$$

we may regard T, which with the c's fixes the state of the system, as fixed by the c's and any one of the i's. Or in general we may take any $(n+1)$ of the $(2n+1)$ variables

$$Tc_1c_2\cdots c_n i_1 i_2 \cdots i_n$$

as arbitrary and consider the others as functions of them.

Let us take the c's as n of the $(n+1)$ independent variables and let us construct n plane diagrams, each with one of the c's as its abscissa and the corresponding i as its ordinate. Whenever the state of the system is modified in any way, some or all of the indicator points will move and trace curves on their diagrams. Whenever the system returns precisely to its original state, each of the indicator points will have traced a closed curve, the area of which represents the work done by that particular force during the cycle. Each of these diagrams will be analogous to the diagram for the case where $n = 1$. The n diagrams will, however, not be independent, since each i is a function of *all* the c's and of T or one of the remaining i's. However, the state of the system is completely defined by the positions of the indicator points, though of the $2n$ variables only $(n+1)$ are independent.

We cannot draw any determinate set of isothermals or adiabatics on any diagram until we have stated how the n variables remaining independent shall vary. We may let the c's all vary proportionately; in that case the value of any one c determines the values of all the rest, and therefore, with T, its own i. Hence, for such a change we have a definite set of isothermals. In general, if we fix the mode of variation of the c's so that we have

$$\left.\begin{aligned} c_1 &= f_1(c_k) \\ c_2 &= f_2(c_k) \\ &\cdots \\ c_n &= f_n(c_k) \end{aligned}\right\}, \qquad (12)$$

any variation of c_k causes a definite motion of each of the indicator points. This amounts, of course, to reducing the number of independent variables from $(n+1)$ to two; but in any case, as said above, the indicator points describe curves which for cyclic processes are closed, and which by their areas represent quantities of

energy of the form $\int idc$. For an adiabatic change the condition is

$$\frac{\delta\epsilon}{\delta T}dT + \sum_1^n \frac{\delta\epsilon}{\delta c_k}dc_k = \sum_1^n i_k dc_k, \tag{13}$$

or

$$\frac{\delta\epsilon}{\delta T}dT = \sum_1^n \left(i_k - \frac{\delta\epsilon}{\delta c_k}\right)dc_k, \tag{14}$$

where $\frac{\delta\epsilon}{\delta T}$ is the heat capacity, α_c, of the system for constant c's. Since $\frac{\delta\epsilon}{\delta T}$, i_k, and $\frac{\delta\epsilon}{\delta c_k}$ are all definite functions of T and the c's, this equation shows how T varies with the c's, and, therefore, how the i's vary. Hence it determines the adiabatic lines on all our diagrams for any given mode of variation of the c's. We see, then, that in any isothermal-adiabatic process each indicator point describes a curve composed of isothermal and adiabatic pieces.

§ 5.

As an illustration of a system of which the state is defined by more than one parameter beside T, we may take a mass of fluid confined in an enclosure of which the volume depends on the positions of several pistons moving in cylinders. To start with, we will neglect gravity, capillarity, and all other outside forces, except the forces on the pistons, and consider the pressure, P, to be uniform throughout the fluid. As our c's we may take the distances of the pistons from certain normal positions. The i_k corresponding to any c_k is the external force needed to keep that piston in equilibrium, or

$$i_k = A_k P,$$

where A_k is the area of the piston in question. The state of the system is determined by T and the c's. Or we may take the c's and any one of the i's, since any one of the i's determines P, and hence all the other i's, as well as T. The illustration as given is imperfect, for we cannot take more than one of the i's as an independent variable, because there is a necessary connection between the i's which makes them all functions of one. In other words, we do not in this case have the equations (10) as *independent*

equations, and the i's do not determine the c's. But we may make our illustration complete by ceasing to disregard gravity. In that case, by reason of the hydrostatic pressure, each i becomes a function of all the c's and of T. We do, therefore, have the equations (10); the c's determine the i's; the i's determine the c's completely, and we may take any $(n+1)$ of the $(2n+1)$ variables

$$Tc_1c_2\cdots c_n,\ i_1i_2\cdots i_n$$

as independent.

§ 6.

If dQ_k is the heat taken in by the system during a change, dc_k, while T and all the other c's remain constant, we may write

$$dQ_k = d\epsilon_k - dw_k, \tag{15}$$

or

$$dQ_k = \frac{\delta\epsilon}{\delta c_k}dc_k - i_k dc_k. \tag{16}$$

If we let

$$d\eta_k = \frac{dQ_k}{T}, \tag{17}$$

we obtain from (16)

$$d\eta_k = \frac{1}{T}\left(\frac{\delta\epsilon}{\delta c_k} - i_k\right)dc_k, \tag{18}$$

and $d\eta_k$ is a definite quantity.

If the system undergoes *any* infinitesimal reversible modification we have

$$dQ = a_c dT + \sum_1^n\left(\frac{\delta\epsilon}{\delta c_k} - i_k\right)dc_k, \tag{19}$$

and

$$\frac{dQ}{T} = \frac{a_c}{T}dT + \frac{1}{T}\sum_1^n\left(\frac{\delta\epsilon}{\delta c_k} - i_k\right)dc_k. \tag{20}$$

If we let

$$\frac{dQ}{T} = d\eta, \tag{21}$$

we get from (20) and (18)

$$d\eta = \frac{a_c}{T}dT + \sum_1^n d\eta_k. \tag{22}$$

Hence if there exists a function, η, of the variables which determine the state of the system, such that

$$d\eta = \frac{dQ}{T},$$

we may consider its variations as due to the variations of all those variables taken separately, and we may speak of the derivatives of η with regard to the separate variables, defining them by the equations

$$d\eta_k = \frac{\delta\eta}{\delta c_k} dc_k, \tag{23}$$

and

$$\frac{a_v}{T} dT = \frac{\delta\eta}{\delta T} dT. \tag{24}$$

§ 7.

It remains to be shown that the function η, defined (except, of course, for an arbitrary constant) by (21), does exist. For this we have only to show that for any reversible cycle

$$\int \frac{dQ}{T} = 0.$$

We have shown already that for any cycle of the whole system each indicator point describes a closed curve. By substituting for the actual cycle one composed of an infinite number of isothermal and adiabatic elements, differing only infinitesimally from the actually performed cycle, we shall substitute for the curve drawn by each indicator point a broken line composed of isothermal and adiabatic elements, and differing in position from the original curve only infinitesimally. We can thus break up our cycle, as usual, into an infinite number of cycles, each of which is composed of two isothermal and two adiabatic changes, and each of which can be performed separately. Each of these infinitesimal cycles has its n representations on the n plane diagrams. Each cycle may be performed in connection with a Carnot engine working between the same temperatures, and we get by the ordinary reasoning,

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0, \tag{25}$$

or

$$\int \frac{dQ}{T} = 0.$$

The cycle which we substituted for the actual cycle is equivalent to the sum of all these infinitesimal cycles. This is seen by re-

ferring to the separate diagrams, where we have all the lines, except those forming the outside boundary, traversed once in each direction, so that the quantities of work and heat involved vanish. We therefore have for the substituted cycle

$$\int \frac{dQ}{T} = 0.$$

For the actual cycle the curves on the n diagrams differ in area by only infinitesimal quantities from the curves for the substituted cycle. The two cycles differ only in respect to infinitesimal quantities of energy, since an area on any diagram means a quantity of energy. For each of the infinitesimal cycles formed by the contours of the actual and the substituted cycles we have $T =$ constant, $\int dQ = \frac{1}{\infty^2}$, and therefore

$$\int \frac{dQ}{T} = \frac{1}{\infty^2}. \tag{26}$$

For the whole number we have

$$\int\int \frac{dQ}{T} = \frac{1}{\infty}. \tag{27}$$

Therefore for the actual cycle, if we neglect the infinitesimal quantity (27), we have

$$\int \frac{dQ}{T} = 0.$$

It follows that there exists a function η, such that for all reversible modifications of the state of the system

$$d\eta = \frac{dQ}{T}.$$

We may therefore use the entropy and its derivatives

$$\frac{\delta\eta}{\delta T}, \quad \frac{\delta\eta}{\delta c_1}, \quad \frac{\delta\eta}{\delta c_2} \cdots \frac{\delta\eta}{\delta c_n},$$

as perfectly well defined quantities in the consideration of any reversible change in the state of a system which is given by any finite number $(n + 1)$ of variables

$$T c_1 c_2 \cdots c_n.$$

Note.— It is perhaps worth while to remark that though the mode of variation of the c's is not in general unimportant for the graphic representation, since we cannot actually construct any diagram for itself independently of the others, yet the change of η depends only on the total changes of T and the c's. For after fixing the mode of variation of the c's as the system passes from a state I. to a state II., we may make the modification in passing back from II. to I. in any way we please (so long, of course, as all the changes are reversible).

THE REFRACTIVE INDEX AND REFLECTING POWER OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.[1]

BY A. D. COLE.

THE electromagnetic theory of light as originally given by Maxwell included only the optics of a single wave-length, and required extension to justify and harmonize the facts of selective absorption and dispersion. This extension was made by Helmholtz. According to him a change in the refractive index with the wave-length is to be expected throughout the complete spectrum; but beyond a certain wave-length these values approximate rapidly to a fixed limiting value, so that the index may be regarded as practically constant for longer waves. The period of this characteristic wave-length is theoretically of the same order of magnitude as that of the slowest possible vibration of the molecule, but it is practically unknown. Yet recent researches render it probable that this close approximation to a constant limiting value is exhibited by wave-lengths much smaller than all electrical waves thus far produced. H. Rubens has shown[2] that several substances which fail to satisfy the fundamental relation of Maxwell, $\mu^2 = K$ (where μ is the refractive index and K the dielectric constant of the substance), for light waves, do fulfil it for radiations in the infra-red only a little below the visible spectrum. For some other substances, — resin oil, olive oil, and certain kinds of glass, — Arons and Rubens[3] have shown that this relation is satisfied with electrical oscillations of 6 meters' wave-length, although not with light waves. E. Cohn[4] has further shown that even with water, whose refractive index for light differs so much from the square root of the dielectric constant, the index for long electrical

[1] Author's abstract from Wiedemann's Annalen der Physik und Chemie. Bd. 57, p. 290, 1896.

[2] Wied. Ann. 45, p. 238, 1892.

[3] Arons u. Rubens, Wied. Ann. 42, p. 581, 1891.

[4] E. Cohn, Wied. Ann. 45, p. 370, 1892.

waves conforms to the law of Maxwell. By entirely different methods Ellinger[1] and Udny Yule[2] have reached similar results. It seems probable that the great change in the value of the refractive index shown by these substances occurs in a part of the spectrum much nearer light-waves than the Hertz oscillations. Still, the researches of Grätz and Fomm[3] and of Drude[4] indicate a certain amount of dispersion even for long electrical waves. It seemed desirable, therefore, to investigate the refractive index of these substances, particularly of water and alcohol, to see if it is the same for very short electrical waves as for long ones.

It seemed first advisable to repeat the measurements of the index for long waves by the method of Cohn with modified apparatus, and to extend the method to alcohol. It will be unnecessary to give more than a very brief account of these experiments, since P. Drude has published the results of a similar research since our work was finished.

I. Velocity of Long Electrical Waves in Water and Alcohol by the Method of E. Cohn.

The apparatus used was very similar to that used by Cohn. An induction coil developed oscillations in two zinc plates 40 cm. square, with attached rods separated by a spark gap. Each of two smaller plates, in front of these, was attached to one end of a long horizontal wire. These wires were stretched parallel, 2.5 cm. apart. They extended several meters through air and then passed through rubber stoppers in the end of a glass trough 68 cm. long, and through the whole length of the trough. The inside of this glass trough was coated with pure silver by chemical deposition. At the inner trough end, and at least two positions outside, bridges of wire were placed across the parallel wires, the proper location being determined when a maximum effect was noticed in two little "Leyden jars" halfway between adjacent bridges, as shown by a Paalzow-Rubens dynamo-bolometer[5] in connection with a re-

[1] Ellinger, Wied. Ann. 46, p. 513, 1892.
[2] Udny Yule, Wied. Ann. 50, p. 742, 1893.
[3] Grätz u. Fomm, Wied. Ann. 54, p. 626, 1895.
[4] P. Drude, Wied. Ann. 54, p. 352, 1895.
[5] A. Paalzow u. H. Rubens, Wied. Ann. 37, p. 769, 1889.

flecting galvanometer. The "Leyden jars," as in the researches of Rubens,[1] consisted of short pieces of thick-walled glass tube threaded on the wires and surrounded by a single turn of fine wire as "outer coating." A second pair of "Leyden jars" was placed within the liquid in the trough, and served to show resonance when the wire bridges were brought into proper positions. These had threads of mercury as outer coatings, insulated by glass from the liquid in the trough. The research differed from that of Cohn in the following respects:

(1) Metallic containing vessels — one of zinc and one of silvered glass — were used. These were provided with metallic covers, as Rubens and Arons have shown that this is important with some substances. For water and alcohol, however, it seems to make little difference, as measurements with the glass trough unsilvered gave about the same result.

(2) The containing vessels were much smaller — from 3.5 to 10 liters capacity, instead of 50. This was made possible by having the parallel wires only 2.5 cm. apart, instead of 7 cm.

(3) The use of at least three bridges outside, and as many inside, gave opportunity to eliminate the influence of the trough end. Cohn assumed that the correction of the wave-length in air due to this cause was the same as for that in the liquid. But this was proved untrue experimentally for our apparatus, and so the values of the waves next the trough end were not used in the calculations.

(4) Four bridge positions were found within the trough of water instead of only two.

The results for distilled water are presented in tabular form. The letters λ_a and λ_w denote half-wave-lengths in air and water respectively, $\frac{\lambda_a}{\lambda_w}$ = ratio of velocities = μ, the refractive index.

The experiments with the silvered trough were much more carefully performed than the others, and the value 8.95 for 19° C. cannot be far wrong. P. Drude has obtained 8.7 for waves 60 cm. long, and Cohn and Zeemann, 8.93 to 8.99.

In similar measurements with commercial absolute alcohol more

[1] H. Rubens, Wied. Ann. 41, p. 154, 1890.

difficulty was experienced in getting good resonance and well-defined maxima. Only two bridge positions within the liquid could be obtained. The alcohol seemed to have higher absorption

Trough used.	λ_a.	λ_w.	$\mu = \frac{\lambda_a}{\lambda_w}$.	Temp. C.
Glass 68 × 7.5 × 7 cm. . . .	261.3	27.8	8.5	18°
Zinc 100 × 10 × 10	299.7	33.5	8.9	16–18°
Glass silvered 68 × 7.5 × 7 cm.	155.7	17.1 17.6 } 17.4 17.4	8.96	18–20°

for waves of the length used than even quite impure water. Drude has noted the same difficulties. For waves whose half-length in air was 129.4 the refractive index 5.24 at 18° was obtained. Drude found a smaller number, 4.74.

II. Refractive Index for Short Electrical Oscillations.

The experiments just described demonstrate that the refractive index of water for waves of 300 cm. to 600 cm. is practically constant, and those of Drude carry the limit of constancy down to 60 cm. Our results for alcohol give a higher value for alcohol than his, as we have seen. This may be because we used waves four or five times as long as his. The difference, however, is too small, and the method too uncertain, to justify the inference that the index varies with the wave-length. Later experiments with the short-period oscillations first produced by Righi[1] form a surer basis for such a conclusion.

I first tried to measure the deviation of Righi waves 5 cm. long, produced by a hollow prism, with sides of thin plate glass 15 × 25 cm. and 10° refracting angle. Although measurements could be made in this way with resin oil, the method failed for water and alcohol. The loss of energy by absorption and reflection was so

[1] A. Righi, Rendiconte della R. Accad. de Lincei 2. 1 Sem (11), p. 505.

great that no sparks could be seen in a Righi resonator on the other side.

Next an attempt was made to produce a more sensitive receiving apparatus. Klemenčič[1] had used a thermopile resonator for long waves. This arrangement on a greatly reduced scale gave most excellent results in our work. In Fig. 1, *A* and *B* are bits of copper foil 10 mm. long by 1.5 mm. wide and 4 mm. apart. To each is soldered a fine wire 4 mm. long and bent to an L shape. One is of iron, 0.02 mm. in diameter, the other of German silver, 0.08 mm. in diameter. The wires cross in the middle and are attached to weak copper springs, which maintain a gentle pressure at the point of contact. These springs are connected to a low-resistance reflecting galvanometer. This resonator is mounted on a light wooden frame at the focus of a concave parabolic mirror made of cardboard, covered with three rows of tinfoil strips, 24 mm. long, placed parallel to the focal line of the mirror. As the thermo-element proved very sensitive to heat radiation, it and its mirror were permanently enclosed in a cardboard box, a material which experiment had shown was perfectly transparent to the electrical waves.

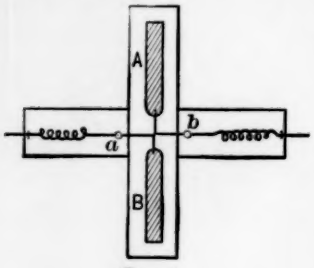


Fig. 1.

A Righi exciter 1 meter away produced galvanometer deflections of 100 to 200 scale divisions. Experiment proved, as theory requires, that deflections were proportional to the energy of the oscillations. Thus the deflection was four times as great when the resonator was 25 cm. distant from the exciter as when it was 50 cm. away. Thus quantitative results were obtainable.

But again the prism experiments failed. A deflection of 100 vanished completely when a prism of water or alcohol was introduced. A flat cell with a layer of water 11 mm. thick allowed only 2 per cent to 3 per cent of the energy to pass. Another with a water film only 0.4 mm. thick showed these results:

[1] Klemenčič, Wied. Ann. 45, p. 62, 1891.

Empty cell, 65 per cent; filled with alcohol, 57 per cent; with water, 32 per cent.

As the reflection of 5 cm. waves on a film 0.4 mm. thick can be only very slight, even for a liquid with refractive index 9, the large energy loss must be due, directly or indirectly, to absorption.

This experience led to the abandonment of the prism experiments and an effort to measure the refractive index for electrical oscillations by an indirect method. The possession of a receiver capable of giving quantitative results suggested the use of Fresnel's formulas connecting reflected energy with refractive index. Their applicability involves the fulfillment of the condition that the square of the absorption coefficient of the substance be small compared to $(\mu - 1)^2$, where μ is the index, a condition which is certainly met by water and alcohol. Further, for water the independence of the reflecting power and the absorption coefficient was established by experiments in which the latter was made to vary through wide limits without affecting the former materially.

It was not convenient to compare directly the amounts of direct and reflected energy, but it was practically simpler to compare the reflecting power of the substance studied with that of a metal surface, and this amount with that received directly.

Reflecting Power of a Metal for Electrical Waves.

The exciter used by Righi for developing electrical oscillations of short period can be used only in a vertical position. This made another form necessary, and that in Fig. 2 was finally adopted. A and A' are glass tubes 5 cm. long. Their inner ends support two bits of brass wire 8 mm. long, with ends rounded. These glass tubes are held by corks in a wider T-tube, as shown, and allow the distance between the bits of brass to be varied. Two copper wires of 1 mm. diameter, their ends bent into a round loop, are held by the friction of little springs centrally in the glass tubes. The outer ends of these wires are joined to the terminals of an induction coil, whose discharge thus has three spark gaps. The T-tube

is filled with heavy petroleum oil, so that the middle spark gap is in oil, the others in air. The tube may be turned about the axis *ab*. A bit of capillary tube *c* allows the gases formed by the decomposition of the oil to escape. For other positions the gases escape through *E*, which is an elbow, connected by a ring of rubber tubing with the third limb of the T-tube. The whole is mounted on a wooden support attached to a spherical concave mirror, so that the middle spark gap remains at the focal point of the mirror when the latter is inclined. The energy radiated was found to vary only slightly when the lengths of the air-gaps were changed, but was much influenced by the length of the oil-gap.

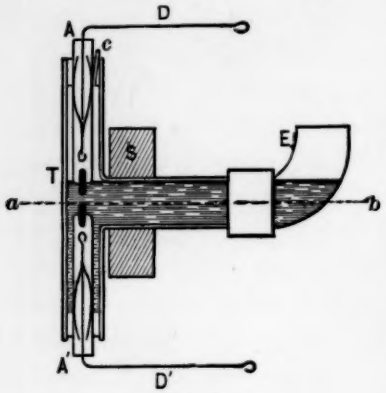


Fig. 2.

Figure 3 shows the arrangement employed to measure the reflecting power of a zinc plate. *C* is the spherical mirror, 50 cm. in diameter, at whose focus *E* the exciter was placed. At a distance of 25 cm. was a pin about which revolved a wooden arm 30 cm. long. At its end the receiving apparatus already described was placed. After four observations of the direct radiation along the axis of the mirror at *R*, the wooden arm was revolved 90° to *R'*, the movable zinc plate *M*, 40 cm. square, placed at an angle of 45° to the axis, and four readings of the reflected energy taken. A series comprised 24 to 32 readings, taken by fours in each position alternately. Three series were taken with the oscillations perpendicular to the incident plane, and eight for those parallel to it. The results were as follows:

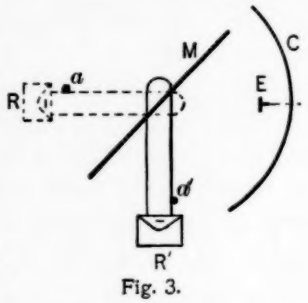


Fig. 3.

Oscillations perpendicular.	Oscillations parallel.
99.5	87.9
101.1	93.1
100.1	88.8
	93.2
	92.9
	95.9
	93.8
	93.5
Mean . 100.2	Mean . 92.4

Thus a metal surface at 45° reflected the radiation without loss if the oscillations were perpendicular to the incident plane, but with about 8 per cent loss if the oscillations were in the incident plane.

Reflection of Electrical Oscillations by Liquid Surfaces.

The axis of the exciter with its mirror was inclined 45° downward, and that of the receiver 45° in the opposite direction. At the intersection of these axes the surface of the liquid to be studied was placed, the liquid being contained in a flat developing tray, 44 × 36 cm. On account of the strong absorption of water and alcohol for these short waves, no interference with reflections from the bottom of the tray was to be feared. A screen of metal, 40 cm. square, was placed between exciter and receiver to cut off any direct radiation. Supports were provided to hold a plate of sheet zinc just above the liquid surface. Three observations of the energy reflected from the metal surface were made, then the metal removed, and an equal number made for the radiation reflected from the liquid. Twenty to thirty readings constituted a series. Five series for oscillations perpendicular, and four for those parallel to the incident plane, were made. An example shows the kind of results obtained. The numbers are galvanometer readings in scale divisions.

Oscillations perpendicular.		Oscillations parallel.	
Water surface.	Metal surface.	Water surface.	Metal surface.
31.0	41.0	18.5	33.0
31.0 →	41.0	18.5 →	32.0
30.0 ↘	43.0	19.5 ↘	30.0
31.0	42.0	18.5	32.5
33.0 →	43.0	19.5 →	35.0
30.0 ↘	45.0	21.0 ↘	31.0
30.0	40.0	17.5	31.5
31.0 →	43.0	18.0 →	32.0
31.0 ↘	43.0	18.0 ↘	27.0
29.0	40.0	15.5	
33.0 →	43.0	19.0	
30.0	43.0	17.5	
Mean 30.8	42.3	18.4	31.6

Whence

$$\text{reflecting power (perpendicular)} = \frac{30.8}{42.3} = 73 \text{ per cent},$$

and

$$\text{reflecting power (parallel)} = \frac{18.4}{31.6} \times 0.924 = 53.7 \text{ per cent},$$

0.924 being the proportion of the total radiation falling upon it that was reflected by the metal plate at 45°.

The results of the several series are collected in the following table:—

Oscillations perpendicular.	Oscillations parallel.
73.6 %	45.6 %
73.9	53.7
69.8	57.1
73.0	54.6
71.8	
Mean . 71.8 %	Mean . 52.7 %

These values were inserted in the two Fresnel formulas for rays polarized in the two planes,

$$(u_{par})^2 = \left(\frac{\sqrt{\mu^2 - \sin^2 i} - \cos i}{\sqrt{\mu^2 - \sin^2 i} + \cos i}\right)^2, \qquad (1)$$

and

$$(u_{perp})^2 = \left(\frac{\mu^2 \cos i - \sqrt{\mu^2 - \sin^2 i}}{\mu^2 \cos i + \sqrt{\mu^2 - \sin^2 i}}\right)^2, \qquad (2)$$

where i is the incident angle 45°, μ the refractive index, and u the amplitude of the oscillations. If the electrical oscillations are perpendicular to the plane of polarization,

R parallel $= (u_{perp})^2$,

and R perpendicular $= (u_{par})^2$.

From equation	(1)	$\mu = 8.8$
	(2)	$\mu = 8.9$
	Mean .	8.85

These results were obtained with distilled water, but the addition of conducting substances, even 50 cc. of H_2SO_4, did not change the amount of energy reflected, although the absorbing power must have been greatly increased. Hence the absorbing power of pure water cannot be great enough to render the Fresnel formulas inapplicable.

Thus practically the same value was found for the refractive index of water for 5 cm. waves as for those 150 cm. to 300 cc. in length. But this was not true of alcohol. Nine series of observations with it gave these results: —

Oscillations parallel.	Oscillations perpendicular.
39.7 %	13.6 %
35.3	17.5
44.3	14.5
42.0	16.2
41.4	
Mean . 40.5 %	Mean . 15.4 %

Inserting these values

Formula (1) gives $(u_{par})^2 = R_{perp} = 15.4$; $\mu = 3.25$

$(u_{perp})^2 = R_{par} = 40.5$; $\mu = 3.15$

Mean $\mu = 3.2$

Thus the two formulas give the same value for μ, but this value is much lower than that obtained for long waves. This suggests

that alcohol possesses strong dispersion for radiation of the wave-lengths used. Apparatus was partially completed for studying oscillations of 1 cm. and of 8 cm. wave-length, but time was lacking to complete the research. We hope to be able to resume the work soon.

The results obtained were so unexpected that the method was tested by applying it to a liquid whose refractive index had been shown to be the same for long electrical waves as for light waves. Refined petroleum (kerosene oil) was taken. A deep vessel was used to avoid disturbing reflections from the bottom. The energy reflected at 45° for oscillations perpendicular was 9.2 per cent, corresponding to $\mu = 1.5$, a value which agrees well with those for light and for long electrical waves.

In conclusion we gather the principal results into one table.

Substance.	Reflecting power.		Refractive index.	
	Oscillations I.	Oscillations II.	Formula (1).	Formula (2).
Metal (zinc) . .	100.0 %	92.4 %	—	—
Water	71.8	52.7	8.8	8.9
Alcohol	40.5	15.4	3.2	3.1
Refined petroleum	9.2	—	1.5	—

Summarizing all the results, we find that

(1) For oscillations of 300 cm. to 600 cm. total wave-length the refractive index for water is 8.95, for alcohol 5.20.

(2) For oscillations of 5 cm. wave-length the two Fresnel formulas give the same value for the refractive index from observations of energy reflected at 45°. For water this value is 8.85, for alcohol 3.2.

(3) The refractive index of alcohol is decidedly greater for long than for short electrical oscillations.

I desire to express my thanks to Dr. Rubens, of the University of Berlin, for numerous suggestions and other help during the whole course of the investigation, and also to Professor Warburg, in whose laboratory the work was done, for his interest and advice.

MINOR CONTRIBUTIONS.

A New Electrolytic Generator for Oxygen and Hydrogen.

By W. S. Franklin.

During the year 1895 a preliminary test[1] of a generator, of novel design, for the rapid and efficient generation of oxygen and hydrogen by electrolysis has been carried out in the Physical Laboratory of Iowa Agricultural College. The earlier part of this test brought out, as was to be expected, a number of essential imperfections in the preliminary design, which was accordingly modified, and the apparatus twice reconstructed. The experimental work was unexpectedly cut short in October, 1895,

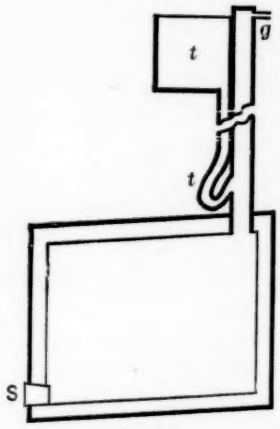


Fig. 1.

Fig. 2.

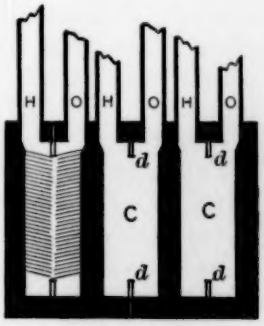


Fig. 3.

before the finally constructed generator was fully tested. The final design, and the more important results of the preliminary tests, are here given. The work is to be continued during 1896.

The generator consists of a number of massive frames cast of an alloy of 25 parts lead and 1 part antimony. The side view of one of these frames is shown in Fig. 1, and a sectional view in Fig. 2. These frames are bolted together, as shown in Fig. 3, the frames being separated by insulating

[1] By Messrs. C. R. Cave and D. M. Hosford. Thesis for 1895. MS. in the College Library, Ames, Iowa.

diaphragms, *dd*, of gum rubber sheet of the kind used for steam packing. The central portions of these diaphragms are cut out as shown in Fig. 4. Two tall lead pipes and one reservoir and trap, *tt*, are fixed to each frame, as shown in Figs. 1 and 2. The water-tight compartments, *cc*, Fig. 3, filled with dilute sulphuric acid (Sp. Gr., 1.17), are broken up by a number of thin glass strips supported at the ends in hard rubber posts, Fig. 5.

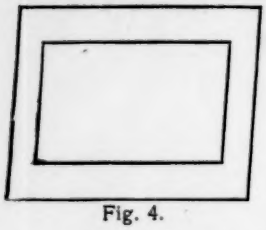


Fig. 4.

Fig. 5.

These glass strips, shown only in one compartment in Fig. 3, prevent the mixing of the two gases. The electric current enters the apparatus at one end-frame and leaves it at the other. The gases collect in the tall lead pipes, pass out through glass tubes attached at *g*, Fig. 1, and are conducted to lead pipes leading to the gas holders. An opening, *S*, Fig. 1 (two in each frame), closed with a rubber stopper, permits of the easy emptying and rinsing of the compartments. The generator is so supported that a vessel may be placed under these openings. The generator in the Physical Laboratory at Ames, now partly finished, is arranged in batteries of five cells each. Each battery weighs about three hundred pounds. The lead frames are 25.5 × 20.4 × 8 cm. outside dimensions, the metal is 1.8 cm. thick, and the other dimensions are as shown in the figures. The openings in the rubber diaphragms are 13 × 19 cm. Such a battery of five cells will stand a continued current of considerably more than 22.5 amperes, without overheating, requiring about 3.8 volts of E.M.F. per cell. The gases produced are not perceptibly mixed.[1]

In the preliminary tests a battery of five cells of the above dimensions was used. *The holes in the diaphragms were only* 7.5 × 15.2 *cm., and no glass strips were used to prevent mixing of gases.*

With this battery the E.M.F. per cell, the final temperature rise above room temperature, and the purity of the gases were determined for currents ranging from 5.9 amperes to 22.5 amperes. The results are exhibited in the following table: —

[1] This point, alone, has been tested with the finally constructed apparatus. The estimates as to heating and required E.M.F. are based upon the preliminary tests. See below.

Number of run.	Current (amperes).	E.M.F. per cell (volts).	Temp. rise (centigrade).	Purity of H (per cent).	Purity of O (per cent).
1	5.9	3.0	2°.5	91.6	75.0
2	7.6	3.0	2°.0	92.7	78.2
3	8.8	3.2	6°.5	93.9	83.0
4	11.2	3.4	13°.5	94.4	83.2
5	13.6	3.5	15°.5	94.8	85.0
6	17.3	3.6	16°.5	94.8	85.0
7	20.0	3.7	21°.8	94.8	85.5
8	22.5	3.8	24°.0	94.7	86.2

The diaphragms for the first battery of five cells which was set up were made of best quality hard rubber sheet, 1.5 mm. thick. After about a month these diaphragms were found to be very badly broken. The battery was then reconstructed, using gum rubber sheet 1.5 mm. thick, as mentioned above.

The efficiency of a hydrogen and oxygen generator requiring E volts per cell is calculated as follows: During the passage of one coulomb of electricity EQ joules of energy are expended, and .00001039 Q grams of hydrogen, and a corresponding quantity of oxygen are generated. The recombination of this H and O would give $34200 \times 4.2 \times .00001039 \times Q$ joules; 34200 gram-calories of heat being generated by the combustion of 1 gram of hydrogen in oxygen, and 1 gram-calorie being equal to 4.2 joules. The efficiency is therefore $\eta = 34200 \times 4.2 \times .00001039 \times Q \div EQ$, or

$$\eta = \frac{1.45}{E}.$$

Therefore a cell used under such conditions as to require 3.8 volts has an efficiency of 38.2 per cent. The generator described[1] by E. L. Nichols and G. S. Moler required, under the conditions of their test, 19 volts per cell, the current through each cell being 24.6 amperes. The great efficiency of the generator here described is due to its low resistance; the anodes and kathodes have each 500 qcm. area, and the distance between them is 6 cm.

One great advantage of a low E.M.F. is that it is impossible for gas to be generated upon the detached sediment of PbO_2, which is gradually formed in the cell. The rapidity of formation of this sediment is, however, not great enough to seriously limit the life of the generator; for during the passage of 1240 ampere-hours through the generator, which was tested, the

[1] PHYSICAL REVIEW, Vol. I., p. 373.

amount of PbO_2 formed was not great enough to be estimated. The test was continued at frequent intervals for more than a month, so that it seems that no serious corrosion of the frames could take place in less than fifteen or twenty years of use.

At 15 cents per kilowatt-hour for electrical energy, 660 liters H and 330 liters O would cost about 80 cents, counting 4 volts per cell of generator. This is sufficient to run a strong lime light for three hours. Counting oxygen only as of value, its cost would be $2.40 per kiloliter (about 8 cents per cubic foot). It seems, therefore, with a generator efficiency of $1.45 \div 4 = 36$ per cent, that the electrolytic generation of H and O for laboratory purposes is not only the most convenient, but perhaps also the cheapest method of production.

AN APPARATUS FOR ILLUSTRATING THE LAWS OF FALLING BODIES.

BY H. M. RANDALL AND W. A. MARKEY.

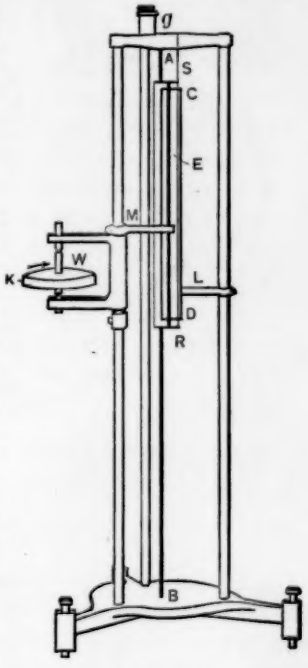
THE problem of uniformly accelerated motion as illustrated by falling bodies has always been a difficult one for laboratory experiment. The piece of apparatus here described was devised for the purpose of obtaining the necessary data from a practically freely falling body, and is simple both in idea and in manipulation.

The apparatus is shown in the accompanying figure and will be seen to possess considerable stability, the top and the base being of cast iron held rigidly together by three pieces of steel shafting. The falling body is quite heavy, and consists of three iron plates, but one shown in the figure, twenty inches in length, soldered to triangular end pieces. Through the centers of these triangular ends passes the guiding rod *AB*. One of the faces of the falling body is furnished with a wide rubber band *E*, which holds in position a strip of stiff paper *CD*. The body is held in suspension at about the height shown in figure by a thread *RS*. This thread is so tied to the rod *R*, which projects from the base of the falling body, that it passes over the forward surface of the paper very near its outer edge. The thread is furnished by the spool *G*, which can be clamped in any position by the thumb screw on its top.

A heavy brass wheel *W* is carefully mounted in an iron frame which turns about the shaft as an axis. The wheel is furnished with a knife edge made from a stout knitting needle. This knife *K* projects about a sixteenth of an inch beyond the circumference of the wheel. The rod *M* is so clamped to the shafting that it arrests the motion of the iron frame when turned toward the suspended body at such a point that the circum-

ference of the wheel just escapes the outer edge of the paper strip. The knife is then adjusted so that if the wheel was rotated it would touch the paper just before it would cut the thread.

The wheel, drawn back from paper, is set in rapid rotation, in the direction indicated by the arrow, by a heavy cord wound around the upper part of its axle. It is then brought opposite the paper. The knife cuts the thread and at the same time makes a fine cut upon the forward surface of the paper, about an eighth of an inch long, so that the first line is made the instant the body begins to fall. With each succeeding rotation of the wheel the knife makes a fine cut on the edge of the paper. The distances between the cut lines are evidently the distances the body has fallen during the corresponding revolutions of the wheel or units of time. Since the rotation of the wheel, while the body is falling its own length, may be regarded as practically uniform, these units of time are considered equal. The number of records obtained on a strip of paper eighteen inches long varies from six to fifteen, according to the velocity of the wheel.



The body strikes on a pad on the tripod. The paper is taken from under the rubber band, and the distances between the lines are measured. As the knife edge is very sharp, the lines are fine and permit very accurate measurement. The rod *L* is so adjusted that it just touches the back side of the iron plate upon which the paper is fastened, thus preventing any rotation of the falling body that might be caused by the knife.

The apparatus is being used at present in the laboratory, and the results obtained are uniformly good. The following tables express the results obtained in two experiments, the unit of length being the thirty-second part of an inch.

Measured acceleration.	Computed acceleration.	Difference.	Measured S'.	Computed S'.	Difference.	Measured S.	Computed S.	Difference.
	17.54	+0.05	8.9	8.77	+0.13	8.9	8.77	+0.13
17.4	17.54	−0.15	26.2	26.31	−0.11	35.1	35.08	−0.02
17.5	17.54	−0.05	43.7	43.85	−0.5	78.8	78.93	−0.13
17.6	17.54	+0.05	61.3	61.39	−0.09	140.1	140.32	−0.22
17.6	17.54	+0.05	78.9	78.93	−0.03	219.0	219.25	−0.25
17.6	17.54	+0.05	96.5	96.47	+0.03	315.5	315.72	−0.22
17.7	17.54	+0.15	114.2	114.01	+0.09	429.7	427.73	−0.03
17.5	17.54	−0.05	131.7	131.55	+0.15	561.4	561.28	+0.12
18.0	17.76	+0.24	9.0	8.88	+0.12	9.0	8.88	+0.02
17.6	17.76	−0.16	26.6	26.64	−0.04	35.6	35.52	+0.08
17.7	17.76	−0.06	44.3	44.40	−0.1	79.97	79.92	+0.05
17.9	17.76	+0.14	62.2	62.16	+0.04	142.1	142.08	+0.02
18.0	17.76	+0.24	80.2	79.92	+0.28	222.3	222.0	+0.3
17.7	17.76	−0.06	97.9	97.88	+0.02	320.2	319.68	+0.52
17.6	17.76	−0.16	115.5	115.44	+0.06	435.7	435.12	+0.58
17.8	17.76	+0.04	133.3	133.2	+0.1	569.0	568.32	+0.68

The distances the body has fallen during the succeeding units of time are measured and placed in the column headed "Measured S'." Subtracting each distance from the succeeding distance, the values in the first column are obtained, which are the measured accelerations and should be uniform. The average of these accelerations is taken as the computed acceleration in the second column. The third column contains the difference between the measured and computed accelerations. The fifth column contains the distances the body should fall during the succeeding units of time, according to the formula $S' = \frac{1}{2} a(2t - 1)$, where a is the average acceleration. The sixth column contains the difference between the measured and the computed values. The seventh column contains the total distances the body has fallen in the corresponding units of time. The eighth shows the distances the body should have fallen according to the formula $S = \frac{1}{2} at^2$, where a is the average acceleration. The ninth column contains the difference between the measured and the computed values of the total spaces.

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NEW BOOKS.

A Treatise on Bessel Functions. By ANDREW GRAY and G. B. MATHEWS. London and New York, Macmillan & Co.

The purpose of this treatise is to supply such a treatment of the theory of the Bessel and allied functions as shall make them practically useful to physicists. The larger portion of the treatise is therefore devoted to illustrative physical problems. It is, however, very complete in the analytical treatment, and includes some of the latest researches on the theory of the Bessel functions. The authors have shown that almost all of this theory is useful in the solution of various physical problems. Some parts are, however, more instructive on the mathematical than on the physical side, notably the chapter on the complex theory.

The book is neatly printed, well written, and remarkably free from errors. I have noted a few errors of reference, however, and on page 8, line 2, $(2n + s)$ should be $(2n + 2s)$.

The first chapter is devoted to three physical problems in which the Bessel functions first presented themselves, viz., the small oscillations of a flexible chain suspended by one end, the flow of heat in a solid circular cylinder, and Bessel's original problem, the determination of the eccentric anomaly of a planet in terms of the time.

In the second chapter we find the solution of the differential equation that defines the Bessel functions, viz.,

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0.$$

If r be the *lowest* power of x in a series solution of this equation, it is shown that $r^2 = n^2$, so that there are only two such series, beginning respectively with x^n and x^{-n}, where n is positive. These series, with appropriate constant coefficients, are denoted by $J_n(x)$ and $J_{-n}(x)$ respectively, and differ from each other only by the sign of n. When n approaches an integral value, the second series approaches $(-)^n$ times the first series, so that another solution of the differential equation is required when n is integral. This is supplied by either of two functions of the form $J_n(x)\log x$ + an infinite series beginning with x^{-n}.

In following the demonstrations the reader will need to make liberal use of pencil and paper, since only the principal results, and the steps necessary to take to obtain them, are given by the authors. The book is on

this account more valuable for reference, since it is not encumbered with a large number of intermediate equations. The reader will find, also, that in following a piece of work himself with pencil and paper, he will become more quickly familiar with the subject and its leading methods than if he were saved the labor of doing this by having every equation supplied in the text. One finds after reading in this way the second chapter of only ten pages, that he is prepared to enter at once upon the physical problems, only referring to the omitted eight chapters whenever necessary.

In order that the character of the work may be fully realized by physical students, we give outlines of some of the problems which it contains.

The first physical problem considered is that of the small vibrations of circular membranes. The equations of motion are first obtained with respect to rectangular axes, the axis of z being perpendicular to the plane of the fixed circular boundary. These equations are then transformed to cylindrical co-ordinates by the substitutions $x = r\cos\theta$, $y = r\sin\theta$. This is the usual transformation in the various physical problems in order to put them in form suitable for solutions in Bessel functions. The transformations are readily made in the case of u, $\frac{du}{dx}$, $\frac{du}{dy}$, etc., by considering that u is a function of r, $= \sqrt{x^2+y^2}$, and $\theta = \tan^{-1}\frac{y}{x}$, so that

$$\frac{du}{dx} = \frac{du}{dr}\frac{dr}{dx} + \frac{du}{d\theta}\frac{d\theta}{dx} = \frac{du}{dr}\cos\theta - \frac{du}{d\theta}\frac{\sin\theta}{r}, \text{ etc.}$$

After the transformation to cylindrical co-ordinates, the normal types of vibration are sought by putting $z = v\cos n\theta\cos pt$, where v is a function of r only. The resulting equation shows that v is a Bessel function, which becomes, after introducing the boundary conditions, $v = A\cdot J_n(\kappa r)$. The constant κ is such that v vanishes at the boundary, or when $r = a$, the radius of the boundary. This equation for κ has an infinite number of roots for each integral value of n. Each integral value of n thus corresponds to a series of normal types of vibration in which there are 0, 1, 2, 3, . . . , concentric circular nodes within the circular boundary, and n diametric nodes that divide the circle into equal sectors. It is further shown how any possible type of vibration may be determined as a composition of normal types. Annular membranes bounded by concentric circles are similarly treated.

In the chapter on hydrodynamics several interesting problems are solved. The first is a case of steady motion of an incompressible fluid, bounded by two coaxial cylinders $r = a$, $r = b$, and two planes $z = h$, $z = -h$. The motion is such that each particle always moves in a plane through the axis, and possesses a molecular rotation that is inversely proportional to its dis-

tance from the axis. The stream lines are shown to be the intersections of the planes $\theta = \text{const.}$, with the surface $\psi = \text{const.}$, where ψ is Stoke's current function and is independent of θ. The value of ψ is found in terms of functions that have the same relation to Bessel functions that hyperbolic functions have to circular functions. These functions are considered in the chapter on the complex theory, since they are Bessel functions with imaginary arguments. They may be considered independently, however, as solutions of the Bessel differential equation, in which the coefficient $(x^2 - n^2)$ of y is changed to $-(x^2 + n^2)$.

The solution for ψ shows that the stream lines in any plane through the axis consist of the rectangular section of the boundary, corresponding to $\psi = 0$, and of a series of closed curves within the rectangle that are symmetrical with respect to the axis $z = 0$. If c be the maximum value of ψ within this rectangle, then $\psi = c$ is the equation of the circular core or filament of this interesting vortex ring.

The authors next consider the small oscillations of a cylindrical vortex rotating approximately as a rigid body, and a hollow cylindrical vortex having approximately an *irrotational* rotation about the axis ($r^2\theta = \text{const.}$), and also a combination of these two cases. The solution consists in determining the velocity systems, or the radii of the approximately circular but vibratory stream lines, and the varying pressures. Two problems of wave motion and the application of Bessel functions to the two-dimensional motion of a viscous liquid complete the chapter on hydrodynamics.

In the chapter on the steady flow of electricity, the following expression is obtained for the potential at the point (r, z) due to a circular disk of radius r_1, situated at the origin and perpendicular to the z axis on which there is a charge of electricity in equilibrium at potential c,

$$V = \frac{2c}{\pi} \int_0^\infty e^{-\lambda[z]} \sin(\lambda r_1) J_0(\lambda r) \frac{d\lambda}{\lambda},$$

where $[z]$ denotes the positive value of z.

This result leads to the consideration of several important problems, such as the flow of electricity from the disk when immersed in an unlimited conducting medium or laid on the bounding surface of a conductor, or the resistance between such a disk source and sink when placed on a conducting surface at such distance apart as not to interfere with each other. Also the problem of the potential and resistance of an infinite conductor bounded by parallel plane faces, the disk source and sink being applied at opposite points of the face; also when this solid instead of being infinite is bounded by a circular cylinder with elements perpendicular to the plane faces, the disks being situated at the centers of the faces, and the whole surrounded by non-conducting matter. The problem of the resistance due to the pas-

sage of electricity from a large metal plate through a thin stratum of slightly conducting material to a second conductor of relatively smaller conductivity than the first, etc., is likewise considered.

In the chapter on the propagation of electro-magnetic waves along wires, the authors begin with the modified forms of the equations of the electro-magnetic field used by Hertz and Heaviside. These are that the variation $\left(4\pi k + \kappa\frac{\delta}{\delta t}\right)$ of the electric force equals double the rotation of the magnetic force, and that the variation $-\mu\frac{\delta}{\delta t}$ of the magnetic force equals double the rotation of the electric force, where k is the conductivity, κ the electric inductive capacity, and μ the magnetic inductive capacity of the medium. By the rotation of a force at any point is meant the molecular rotation in the hydrodynamic analogue in which force corresponds to velocity. Using dilatation in the same sense, it follows from the above relations that the dilatations of the electric and magnetic forces are each zero. It also follows that the electric and magnetic forces satisfy the same differential equation of propagation. In the case of propagation along a straight wire, the field is symmetrical round the wire at every instant, so that transforming to cylindrical co-ordinates x, ρ, θ, the variable θ disappears, and we have two components of electric force P, R along x, ρ, and one component of magnetic force H, perpendicular to x, ρ. The transformed differential equations of propagation give two equations, one satisfied by P, and the other satisfied by R, H.

The problem considered is essentially that of a cable, viz., a wire of finite radius surrounded by an envelope of insulating isotropic substance, and the whole surrounded by an infinite conducting material. For the insulating envelope $k = 0$, and taking the normal type of simply periodic forces the corresponding differential equations for P and R, H become Bessel equations of zero and first order respectively. In the conductors $\frac{\kappa}{k}$ is supposed small, and similar differential equations are obtained for those mediums. The general values of P in the three mediums being thus found, the values of R, H are found from differential equations connecting them with P, and the determination of constants from the boundary conditions completes the solution. This solution is fully completed for waves of slow vibrations in a cable of small radius. The case of rapid vibrations is also discussed, and the theory of Hertz's vibrator is considered.

The authors have introduced an unusually long chapter on the diffraction of light, with several diagrams and numerical computations, the object being, as stated in the preface, to attract more general attention to the valuable and interesting results contained in Lommel's memoirs. The

Bessel functions are applicable, of course, only to the case of symmetry round an axis, and the specific problem considered is that of the diffraction produced by a small circular opening on which light falls from a point-source that lies on a perpendicular erected at the centre of the opening. The quantity that it is desired to find is the intensity of the illumination at any point of a plane screen parallel to the plane of the opening, corresponding to any given wave-length of source.

Various miscellaneous applications are given in Chapter XV. This chapter might very well be read by the student before the other more difficult chapters on general subjects. A long list of examples for practice also follows this chapter.

The book closes with over 40 pages of tables of the numerical values of the Bessel functions, a short bibliography of three pages, and a page of the graphs of $J_0(x)$ and $J_1(x)$.

From the preceding outline of contents the reader will see that the treatise is one that cannot fail to be of great value to the student of physics. Physical investigations have now become so far advanced that it is necessary for the student who wishes to keep up with his times to be as familiar with several important functions of higher order as he is with the trigonometric and logarithmic functions, and the present work can be relied upon to furnish him with accurate knowledge of all the properties of Bessel functions that are suitable for their practical application. There is, in fact, no other book of this kind in the English language that gives this information in such a direct, compact, and well-arranged form, and with such a wealth of illustrative examples.

A. S. HATHAWAY.

ROSE POLYTECHNIC INSTITUTE.

Physikalische Krystallographie und Einleitung in die krystallographische Kenntniss der wichtigeren Substanzen. Von P. GROTH. Third edition, rewritten. pp. xvi+783. Leipzig, W. Engelmann, 1895.

The first edition of this book appeared in 1876. The second edition appeared in 1885, and was considered at that time to be "the most satisfactory work of its kind in any language." It must be a great pleasure, in expressing one's estimate of such a book as this third edition, to be able to acknowledge an old debt of gratitude to the author.[1] The present writer finds the book simply admirable, and certainly as satisfactory at this time as the second edition was in 1885. It is distinctly a treatise on physical crystallography, and chiefly satisfactory in being such.

[1] See the excellent review of this book of Groth's by Mr. L. Fletcher, F.R.S. Nature, January 30, 1896.

Professor Groth points out that crystal form, inasmuch as it is the result of crystal growth, is essentially a physical property, so that several extensive chapters are rightly devoted to the geometrical properties of crystals. It is in these chapters that the most striking changes from the second edition occur; changes which grow out of the unequivocal adoption of the conception of the *axis of symmetry*.

A very important feature of the work is the manner in which the theoretical portions, *e.g.* the theory of crystal structure and geometrical crystallography, are based upon experimental facts.

The chapter on the theory of the molecular structure of crystals is chiefly of speculative value, inasmuch as very few, if any, experimental facts have ever been foreshadowed thereby; and placed as it is between the chapters on the physical properties of crystals and the chapters on geometrical crystallography, this chapter certainly disturbs the simplicity of the development. The molecular theory can indeed be used as a basis for geometrical crystallography, as many of us know too well. Professor Groth does not so use it, and such use of it must be misleading and unsatisfactory from its necessarily uncertain experimental foundation.

This molecular theory has great value,[1] however, in its simpler aspects, in giving concrete representations of such experimental abstractions as the *law of crystal homogeneity*, the *law of constant angles*, the *law of rational indices*, and the *law of symmetry*. The theory should by all means be used in this connection, especially in a text for students who cannot be sufficiently familiar with real crystals to enable them to dispense with it. Indeed it must be admitted that such an abstraction as the law of rational indices, for example, cannot mean anything to any one except in so far as it is thought of in connection with a geometrical complex which has that property.

Geometrical crystallography has been unsatisfactory heretofore, — although often spoken of as one of the most complete of the sciences! — chiefly because its connection with physical crystallography has been too much ignored. The following is a brief statement of the physical foundation and of the physical signification of geometrical crystallography as one would get it by reading Groth with this point in view: —

(*a*) Every part of a normal crystal is exactly similar in its physical properties to every other part. (Law of homogeneity.)

Remark. — An abnormal crystal is a more or less fine-grained intergrowth of differently oriented crystals.

(*b*) The physical behavior of a crystal is different in different directions. (Law of ælotropy.)

Remark. — A crystal is a body which is homogeneous and ælotropic.

[1] See p. 145, Vol. I., Text-Book of Physics, Nichols and Franklin.

The external form of a crystal is merely an indicator of its growth. (See *c*.)

(*c*) Certain directions in a crystal are physically similar.

Remark. — The number of distinct directions in a given crystal which show similarity, depends upon the physical property which is made use of in testing it. Professor Groth gives a good *résumé* of the physical properties of crystals in this connection (pp. 242–243). In the classification of crystals it is, therefore, important to make use of those physical properties which show always the least number of similar directions in a crystal. Crystal growth and crystal disaggregation, under the action of a solvent or by vaporization, show in every case the least number of similar directions in a crystal. Therefore, two directions in a crystal are admittedly similar when the crystal grows or dissolves similarly in those directions. In some cases the growth of a crystal (indicated, of course, by its form) is similar in two directions, which are shown to be different by dissolution, or *etching*, as it is technically called. Therefore, etching is the ultimate criterion in determining the symmetry (see *h*) of a crystal.

(*d*) During slow growth a crystal becomes a polyhedron, a solid bounded by plane[1] faces. For example, if the faces of a given crystal be obliterated by cutting, they will reappear in their original positions if the defaced crystal is placed in its mother liquor and allowed to grow.

(*e*) The relative amount of deposition of material upon the different faces of a growing crystal is largely determined by *external* circumstances, and cannot, therefore, be taken as an indicator of the physical similarity of the crystal in different directions. *The angles between the various faces of a crystal* (or between a set of similar directions in the crystal) *are, at a given temperature and pressure, invariable.* (Law of constant angles.)

(*f*) *Similar faces of a crystal.* The faces of a given crystal which are similar polygons (polygons of equal angles) are similar. A set of similar faces is called a crystal form. The minimum number of faces in such a set is one, the maximum number forty-eight (a result of *g* and *h*). This criterion of similar faces sometimes fails, inasmuch as etch figures on two faces of such a set are sometimes different. Similar faces may therefore be defined as faces which dissolve similarly.

Remark. — Those directions in a crystal which are similarly situated with respect to similar faces, are similar with respect to every physical property. It is possible to derive, from this fact, a statement of the law of rational indices (*g*) referring to *sets of similar directions* in a crystal.

[1] Some crystals have curved faces. The conception of similar faces applies to such crystals and also the law of rational indices, for this law relates strictly to the directions in a crystal which are physically similar, and cleavage and all physical properties of such crystals are in definite directions.

(*g*) Choose three edges of a crystal as axes of reference. Consider the three intercepts, *a*, *b*, and *c*, on these axes, of a plane parallel to a given face of the crystal. The two ratios $\frac{b}{a}$ and $\frac{c}{a}$, associated with the axes *b* and *c* respectively, are called the indices of the given face. *The respective indices of all faces of a crystal are integral multiples of two parameters.* (Law of rational indices.)

(*h*) Consider a set of similar directions in a crystal. Imagine the crystal to be moved in such a way that this set of directions comes into congruence with its initial position. Such a displacement is called a congruence displacement.

Any translatory displacement is a congruence displacement, from (*a*).

In some crystals a rotation less than 2π about an axis may be a congruence displacement. Such an axis is called an *axis of symmetry*. A screw displacement need not be considered. In the molecular theory of crystal structure, certain translations only are congruence displacements, in which case screw displacements must be considered as involving something not included in independent rotary and translatory displacements.

A set of similar directions in a crystal may in some cases be brought into congruence with their initial position by the ideal process of inversion with respect to a plane, an operation which is exemplified in the formation of an image of an object in a plane mirror. Such a plane is called a *plane of symmetry*.

This, (*a*) to (*h*), is the experimental basis of geometrical crystallography.[1] The elements of crystal symmetry are the *axis of symmetry* and the *plane of symmetry*. Certain combinations only, of these elements, are consistent with the law of rational indices, and all possible combinations may be rigorously derived from that law. This derivation is perhaps not suited to a text of the character and scope of this of Professor Groth's; at least, in his discussion of the thirty-two combinations of axes and planes of symmetry, including the case where neither occur, he does little more than prove the possibility of the combinations without showing that no other combinations are possible.

It is indeed a question whether planes of symmetry do exist.

Any crystal which has at least two axes of symmetry not in the same direction must show symmetry to three mutually perpendicular planes in its thermal and electric conduction, elastic and optic properties (except optic activity), electric and magnetic polarization, etc. Such physical properties from their nonpolar character always lead to the notion of symmetry to planes when there is any symmetry at all.

[1] The claim of Mr. William Barlow, Nature, Nov. 15, 1894, that the conception of Homogeneity pure and simple leads to all the varieties of symmetry presented by crystals, is, in the opinion of the writer, not justifiable.

On the other hand, absence of symmetry to planes is definitely indicated by such physical properties as pyroelectric, piezoelectric, and optic activity, by asymmetric growth, and by asymmetric dissolution or etching. Some crystals, however, show absence of symmetry to planes by etching, but do not show optic activity. Other crystals show absence of symmetry to planes by etching, but to all appearances exhibit symmetry of growth. Whether this is true also for pyroelectric and piezoelectric activities the writer cannot say; but pyroelectric and piezoelectric effects are very difficult to observe, and a failure to detect them would in no case be very significant. Therefore the ultimate criterion of symmetry to planes is by etching, and this test, although in many cases unequivocal, requires in some cases the closest scrutiny to use it with certainty, and in many cases it gives no result at all, and then we have symmetry to planes.

In his development of the theory[1] of the molecular structure of crystals, Sohncke scarcely mentions symmetry to planes. Indeed, point-systems or cell-systems in space can have symmetry to planes only when the elementary points or cells possess that kind of symmetry themselves, and point-systems which are not symmetrical can be built up of such elements. Asymmetric molecules may be grouped in large nuclei (crystal molecules) which are approximately symmetrical, and in the detachment of such approximately symmetrical crystal molecules one would not expect a solvent to produce noticeably asymmetric etch-figures.

From these facts, together with the physical impossibility, not to say utter lack of meaning, of the operation of *inversion*, the writer is inclined to look upon the conception of the plane of symmetry as uncertain, to say the least. Crystal forms which are thought to show symmetry to planes are easily thought of as combination forms. However, the question whether we are to accept planes of symmetry is not very important, because what we know of crystallography at present cannot decide it. The plane of symmetry is a permissible notion and is very convenient.

In regard to Professor Groth's choice of the *composite axis and plane* of symmetry instead of the *center of symmetry*, of which Mr. L. Fletcher (*loc. cit.*) does not see the justification, it may be said that Professor Groth's conception includes the two distinct cases: the composite double-axis and plane, and the composite quadruple-axis and plane, Professor Groth's second and sixth classes of crystal forms respectively; while the conception of the center of symmetry leaves the latter case unprovided for.

Crystallography is certainly growing in importance to the chemist — many chemists would be surprised to know that the theory of the molecular structure of crystals is in almost complete harmony with *all known physical properties of crystals*, and that it includes the whole of stereo-chemistry as

[1] Theorie der Krystallstructur, Dr. Leonhard Sohncke, Leipzig, 1879.

a special case — and no serious student of theoretical chemistry should remain without an intimate knowledge of the subject. It is to be hoped that Professor Groth's promised work on chemical crystallography will soon be in our hands.

During the careful perusal of Groth's book, and of others bearing upon the subject, the writer's thoughts have, perhaps naturally, reverted often to the recent writings on Energetics; in particular to the writings of the great thinker, Ostwald, which have, indeed, a fine *metaphysical* quality. It is with no little pains that one turns from the easy and enticing sweep of such writing to the turgid hardness of such a work as this of Groth's, which is sternly *physical.* Most of us have, indeed, read the former, and many of us await compulsion in taking up the latter. The quality of the advancement of learning is strained! Such easy writings, indeed, "can have only the result: to excite those young in science to dilletantish speculation instead of to deeper study of the masterpieces." We cannot do without geometry yet awhile. Mechanics is geometry! Our theories of light, sound, electricity, and magnetism are geometry! The molecular theory, even in its simplest aspects, is geometry! However, it has been said of the older geometry that "Es ist nur eine ungerechtfertigte Gewohnung Euklidische Geometrie für streng wahr zu halten," and the wisdom of this remark applies to the newer geometries also.

W. S. FRANKLIN.

AMES, IOWA, March, 1896.

Grundriss der Electrochemie. By HANS JAHN. Royal 8vo, pp. 312. Vienna, Alfred Hölder, 1895.

The material contained in this book was presented in a course of lectures at the University of Berlin during the winter semester of 1894 and 1895. It was the reviewer's privilege to hear this lecture course, and to gain inspiration from Dr. Jahn's clear and enthusiastic presentation of the subject. The book, too, is not disappointing in that respect. There is a certain vivacity that adds much to the interest of the reader. The presentation is usually clear, but in a few cases one is at a loss to know what the significance of the symbols is until one has read a half-page farther. Such things occasion much loss of time. A writer should scarcely use *one* symbol in a single topic to express *three* quantities, even if those quantities are such simple expressions as "constant," "concentration," and "temperature."

A knowledge of the differential and integral calculus, and some knowledge of thermodynamics, is essential in order to understand the presentation of the subject. One would also labor through the treatise with difficulty had one no previous knowledge of the subject.

The author may have intended to provide against this latter difficulty, as the first chapter upon "The Foundation Laws of Electrochemistry" is treated in a very elementary manner. The Kohlrausch law, together with the independent wandering of the ions and their absolute velocity, are about the only points in this chapter which would suggest that the reader would soon be introduced to the Arrhenius theory of electrolytic dissociation. This theory is the subject of the second chapter — the longest one in the volume. With this chapter the book proper may be considered to begin, and from this point on one could reasonably call the book a treatise upon electrochemistry based upon thermodynamics. Planck's formula for equilibrium is developed from the second law of thermodynamics. Indeed, all the theories of the new electrochemistry are developed from the principles of energy and entropy. This seems to be a most desirable basis, as the objection which is most frequently raised is that the whole theory is contrary to the doctrine of the conservation of energy. This objection generally comes from persons who are not familiar with the "sharp weapons of mathematical analysis." It suggests the necessity of their acquiring such familiarity, if they would not, as the author says in his preface, "assume the danger of losing all understanding of the developments of theoretical chemistry." The fact, too, that chemists, and not physicists, have generally raised the above objection, confirms the author's statement.

In Chapter II. the tension of the saturated vapor of a solvent in contact with a solution, lowering of the freezing-point of the solvent, osmotic pressure, Ostwald's law of dilution, heat of dissociation, influence of pressure upon conductivity, and the Nernst theory of diffusion are some of the subjects which are given a thermodynamic treatment. Finally we are given Planck's demonstration of current conduction in electrolytes as a mechanical process.

The next chapter treats of the transformation of energy by electrochemical processes. In discussing the mechanical equivalent of heat it is surprising that the determinations made by Rowland, and some of the other more recent and careful determinations, are not mentioned.

The theory of Kelvin and Helmholtz that the chemical and electrical energy of a galvanic cell are equal, is shown to be wrong. That the electrical energy may be greater or less than the chemical energy, according as the temperature coefficient of the cell is positive or negative, is shown by the later Helmholtz equation, an equation developed from the principle of entropy.

The Helmholtz theory of free (*i.e.* available) energy is discussed, and upon it are based some conclusions of great interest concerning the variations of the electromotive force with pressure, and Helmholtz' theory of the variation of current with varying concentration. While this enables

us to understand that a difference of potential exists between different concentrations of the same salt, it teaches us nothing concerning the mechanics of the subject. The theory of Nernst — one of the greatest fruits of the dissociation theory — has given a more satisfactory solution of this question by assuming that the Helmholtz concentration current arises from a difference of ionic concentration and a difference of osmotic pressure in the two solutions. Planck first discussed concentration series as a process of diffusion, and showed from the dissociation theory that the energy appearing as an electromotive force is due to diffusion alone. This theory of Planck is presented, and shown to agree with the results of the above methods.

Air and vacuum electrodes are discussed and the theory of Warburg presented. In a series of the form

$$\mathrm{Hg} \mid \underset{\text{In air}}{\mathrm{H_2SO_4} + \mathrm{XH_2O}} \mid \underset{\text{In vacuum}}{\mathrm{H_2SO_4} + \mathrm{XH_2O}} \mid \mathrm{Hg},$$

the oxygen of the air oxidizes the mercury, which in turn is acted upon by the sulphuric acid. This would bring mercury ions into the solution. The Warburg theory demands that mercury ions must pass into solution at the other electrode, but in much smaller quantity. We may say that all substances are at least sparingly soluble. The theory finds extensive application in the use of platinum electrodes. It explains, too, the capillary electric phenomena which the Helmholtz theory of opposite charges upon the two contact strata left in a rather unsatisfactory state.

The Daniell cell is taken as a type of cells reversible as to the kathion.

The electromotive force is localized at the electrodes as an osmotic process, and at the contact of the electrolytes as a process of diffusion. The author speaks of series reversible as to the anion, and of series reversible as to a kathion and an anion. Following Goodwin, the author attempts to prove that the latter class is essentially reversible in respect to the kathion.

The Lippmann electrometer and the drop electrode method of measuring single potential differences are discussed at some length. Some sources of error of different observers are pointed out. The work and precautions of Paschen are mentioned. This experimenter succeeded in locating forty-five per cent of the electromotive force of a Daniell cell at the contact of the two solutions, which is entirely contrary to the theory of Nernst. One can scarcely help forming the opinion that the drop electrode method of measuring single potential is very uncertain, and that much is left to be learned concerning the action of such electrodes.

Chapter IV. treats of polarization. The variation of free (*i.e.* available) energy at the kathode is discussed, and the conclusion drawn, "that the

kathode polarization is dependent only upon the chemical nature of the separated metal, and the concentration of the solution; and that it reaches a limit equal to the potential difference, demanded by the theory of Nernst, between metal and solution." At the anode the process becomes more complicated; but in the case of strong acids, for which the concentration of the hydrogen ion is nearly constant, the anode polarization is independent of the chemical nature of the ion. After the kathode polarization has reached its limit, the increase of total polarization is to be attributed to an increase in the anode polarization. This takes place rather rapidly until the solution at the anode is saturated with oxygen, when the development of gas begins. From this point the polarization increases very slowly, and there is a more rapid increase of current strength. Polarization in dilute acids, alkaline solutions, and bases, as well as variation of polarization with a change in temperature, are discussed at some length. It follows from the formula, since the concentration of the ions does not enter, that all oxygen acids must have the same polarization.

Only two pages are devoted to accumulators. The author's statement that their action is complicated, and not completely explained, will meet with general approval.

Chapter V., upon the decomposition of some of the most important chemical combinations by means of an electric current, and some applications of electrolysis, concludes the book.

Numerous and valuable tables of data are scattered throughout the book. A small index would have made these tables more accessible for reference. Had such a table been provided, and had numerous references to the original literature been made, this volume would have been very complete.

B. E. MOORE.

The Theory of Light. By THOMAS PRESTON. Second edition. London, Macmillan & Co., 1895.

The present edition of this well-known work differs in general but little from the previous one. Considerably more space has been devoted to the discussion of the rectilinear propagation of light as well as to the explanation of graphical methods. The vibration spiral is considered earlier in the work, although its application is hardly evident until the subject of diffraction is reached.

In the treatment of reflection and refraction at spherical surfaces, the idea of "impressed curvature" has been introduced. While this method is extremely convenient in the development of approximate formulæ, yet the degree of approximation involved is apt to be lost sight of unless the limitation to small apertures is kept constantly in mind.

The discussion of diffraction by graphic methods has been considerably improved. The diagrams showing the methods of strip divisions will doubtless aid the reader in the formation of a clear idea of the meaning of the spirals employed, as well as in better understanding the physical conceptions which underlie the rather complicated analytical expressions usually encountered in that subject. The treatment of the diffraction grating is hardly as complete as might be expected, being limited to rather more perfect gratings than are likely to be produced.

Some additions have been made to the chapter on rotatory polarization, but the greater part of the new matter will be found in connection with the determination of the velocity of light and the subject of relative motion of matter and ether, thus making this portion of the work as complete as one could wish.

The book is, on the whole, quite free from errors or misprints. There are, however, a few, as, for example, on page 93, where the angles are rather confusing.

While the reader will doubtless share the regret of the author that time has not permitted him to "bring all parts of the work up to the standard demanded by the present state of the science" (especially along the lines of the electromagnetic theory), yet it may be expected that the new edition will continue to receive the very favorable consideration extended to the first. It is to be regretted that the author did not append a bibliography of the subject for the convenience of those who may have access to original memoirs only briefly mentioned in the text.

J. S. SHEARER.

Synopsis of Current Electrical Literature for 1895. Compiled by MAX OSTERBERG. pp. 143. New York, D. Van Nostrand Co.

This synopsis is compiled chiefly from the technical journals and magazines for the year, being practically a revision of the synopsis published monthly in *Electric Power*. The synopsis falls midway between an index and a digest, brief mention of the subject-matter of each paper referred to being made, so that the reader may know whether or not to consult the original article.